Chain Rule

$$\frac{d}{dx} f(g(x)) = f'(\underbrace{g(x)}_{y}) \cdot \underbrace{g'(x)}_{\frac{dy}{dx}}$$

$$f(y), \quad g(x)$$

$$\frac{d}{dx} f(g(x)) = \frac{df}{dy} \cdot \frac{dy}{dx}$$

$$T(x,y) = x^2 e^{-y}$$

$$\vec{r}(t) = \langle x(t), y(t) \rangle = \langle t, t^2 \rangle$$

$$\frac{d}{dt} T(\vec{r}(t)) = \frac{\partial T}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial T}{\partial y} \cdot \frac{dy}{dt}$$

$$\frac{d}{dt} T(\vec{r}(t)) \quad \text{at} \quad t = \frac{1}{2}$$

$$\frac{\partial T}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial T}{\partial y} \cdot \frac{dy}{dt} \qquad y(t) = t^2$$

$$\frac{dx}{dt} = 1 \qquad \frac{dy}{dt} = 2t$$

$$\left.\frac{dx}{dt}\right|_{t=1/2} = 1 \qquad \left.\frac{dy}{dt}\right|_{t=1/2} = 2 \cdot \frac{1}{2} = 1$$

$$T(x,y) = x^2 e^{-y}$$

$$x(t) = t \qquad x(1/2) = \frac{1}{2}$$

$$y(t) = t^2 \qquad y(1/2) = \frac{1}{4}$$

$$\frac{\partial T}{\partial x} = 2xe^{-y} \qquad \frac{\partial T}{\partial y} = -x^2 e^{-y}$$

$$\frac{\partial T}{\partial x} = 2 \cdot \frac{1}{2} \cdot e^{-1/4} \qquad \frac{\partial T}{\partial y} = -\frac{1}{4} e^{-1/4}$$

$$\left.\frac{d}{dt}\right|_{t=\frac{1}{2}} T(\vec{r}(t)) = \frac{\partial T}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial T}{\partial y} \cdot \frac{dy}{dt}$$

$$= e^{-1/4} \cdot \underline{1} + \left(-\frac{1}{4} e^{-1/4}\right) \cdot 1$$

$$= e^{-1/4}\left(\frac{3}{4}\right) \approx 0.5841$$

$$T(x,y) = x^2 e^{-y}$$

$$x(t) = t$$

$$y(t) = t^2$$

$$T(x(t), y(t)) = (t)^2 e^{-t^2}$$

$$= t^2 e^{-t^2}$$

$$\frac{dT}{dt} = \frac{d}{dt}\left(t^2 e^{-t^2}\right)$$

$$= 2t\, e^{-t^2} - t^2 e^{-t^2} \cdot (2t)$$

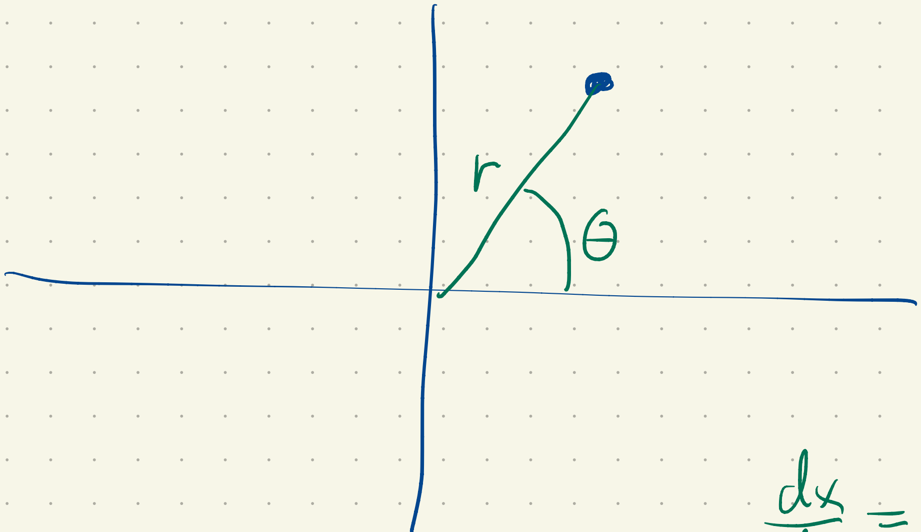
$$= 2t e^{-t^2} - 2t^3 e^{-t^2}$$

at $t = 1/2$ $\quad = 2 \cdot \frac{1}{2} e^{-1/4} - \frac{2}{8} e^{-1/4}$

$$= e^{-1/4} - \frac{1}{4} e^{-1/4} = e^{-1/4} \cdot \left(\frac{3}{4}\right)$$

$f(x,y)$, $x(t)$ and $y(t)$

Suppose we have a curve in the plane but I don't tell you $x(t)$ and $y(t)$ but instead $r(t)$ and $\theta(t)$



$$\frac{dx}{dt} \qquad x = r\cos\theta$$

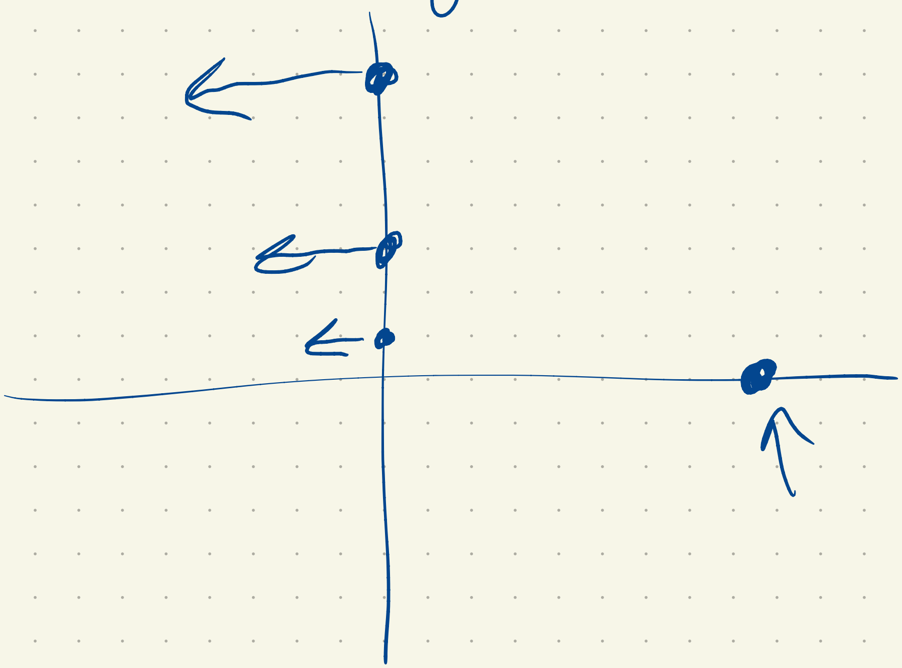
$$\frac{dy}{dt} \qquad y = r\sin\theta$$

$$\frac{dx}{dt} = \frac{\partial x}{\partial r}\cdot\frac{dr}{dt} + \frac{\partial x}{\partial \theta}\frac{d\theta}{dt}$$

$$= \cos\theta\cdot\frac{dr}{dt} - r\sin\theta\frac{d\theta}{dt}$$

$$\frac{dy}{dt} = \sin\theta\frac{dr}{dt} + r\cos\theta\frac{d\theta}{dt}$$

$\theta = \pi/2$



$\theta = 0$

$$\frac{dx}{dt} = \frac{dr}{dt}$$

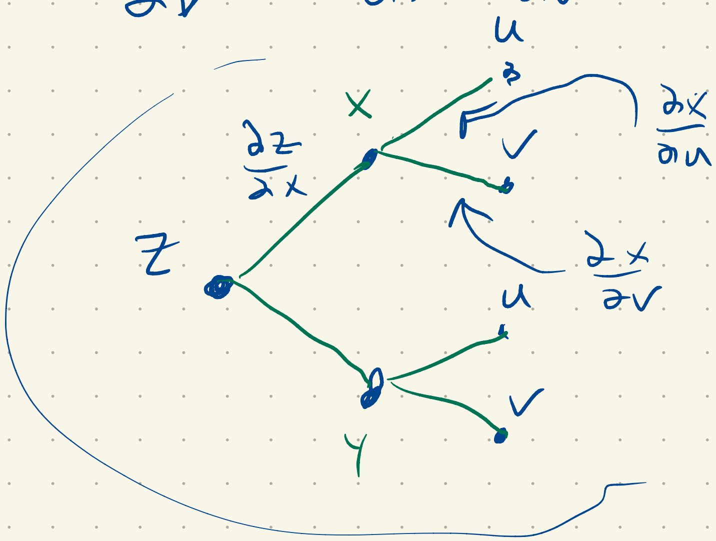
$$\frac{dx}{dt} = 0\cdot\frac{dr}{dt} - r\cdot 1\cdot\frac{d\theta}{dt}$$

$$\frac{dx}{dt} = -r\frac{d\theta}{dt}$$

$$z = f(x,y) \quad , \quad x = x(u,v) \quad y = y(u,v)$$

$$\frac{\partial z}{\partial u} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial u} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial u} \qquad \left[\frac{\partial f}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial f}{\partial y}\cdot\frac{dy}{dt}\right]$$

$$\frac{\partial z}{\partial v} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial v} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial v}$$



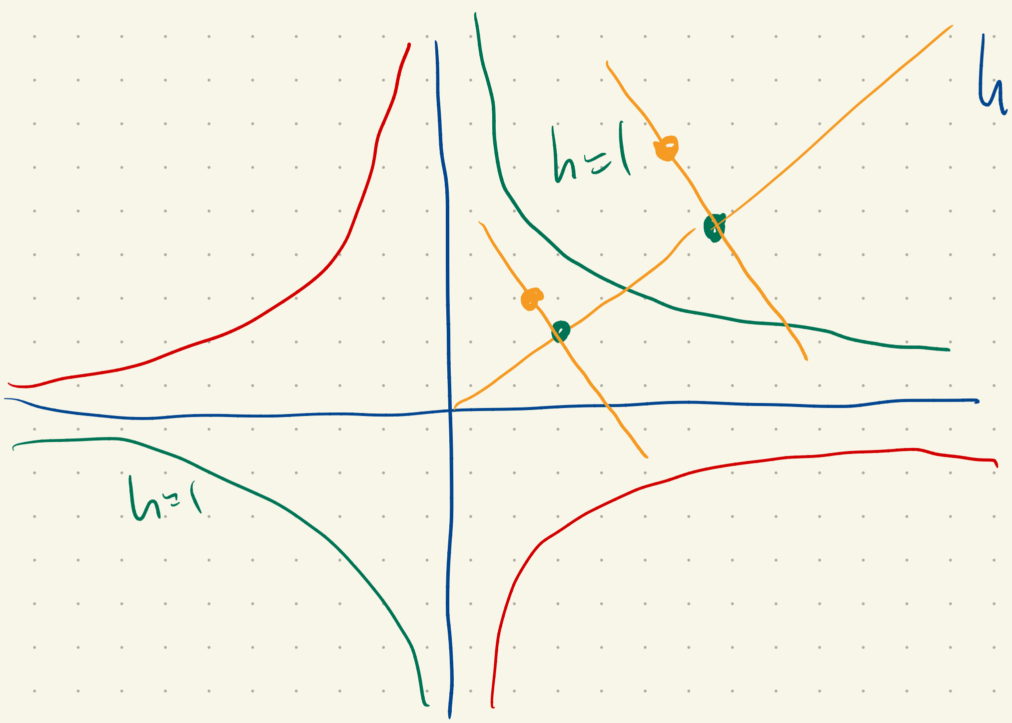
$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial u}$$

$h(x,y)$ height depends on x, y

$h(x,y) = xy$ What is $\frac{\partial h}{\partial r}$? $\frac{\partial h}{\partial \theta}$

$x(r,\theta) = r\cos\theta$

$y(r,\theta) = r\sin\theta$



$h(x,y) = 1$

$xy = 1$

$y = 1/x$

$\frac{\partial h}{\partial r}$

$\frac{\partial h}{\partial \theta}$

$\frac{\partial h}{\partial x}$

$\frac{\partial h}{\partial y}$

$$h(x,y) = xy$$

$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$\frac{\partial h}{\partial r} = \frac{\partial h}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{\partial h}{\partial y} \cdot \frac{\partial y}{\partial r}$$

$$= y \cdot \cos\theta + x\sin\theta$$

$$= r\sin\theta\cos\theta + r\cos\theta\sin\theta$$

$$= 2r\sin\theta\cos\theta$$

$$= r\sin(2\theta)$$

$$\frac{\partial h}{\partial \theta} = \frac{\partial h}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial h}{\partial y}\frac{\partial y}{\partial \theta}$$

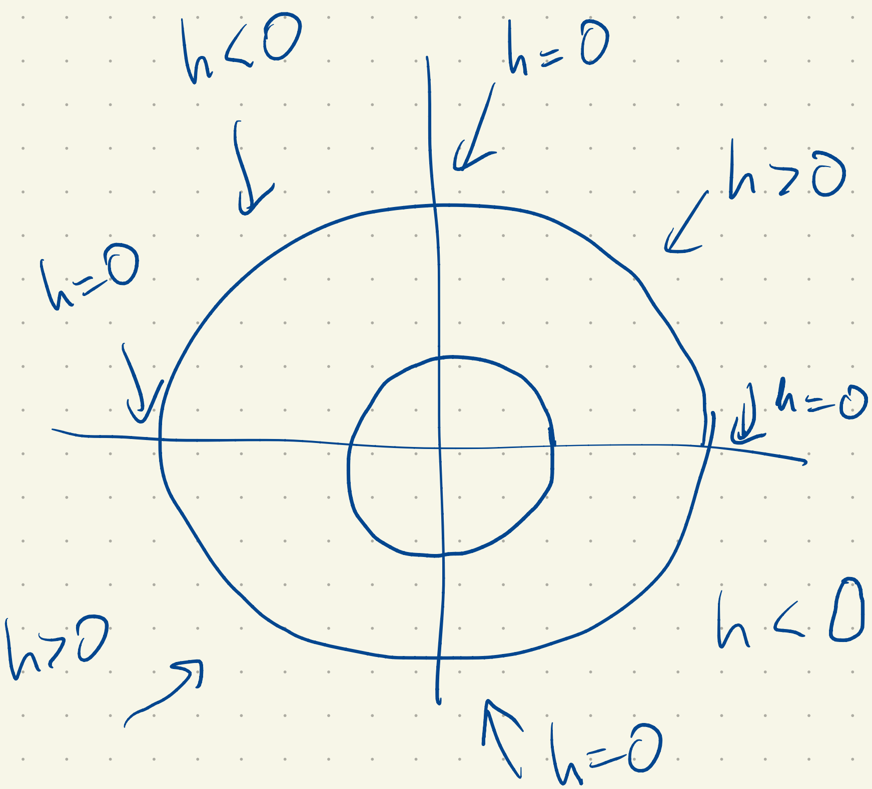
$$= y \cdot (-r\sin\theta) + x\, r\cos\theta$$

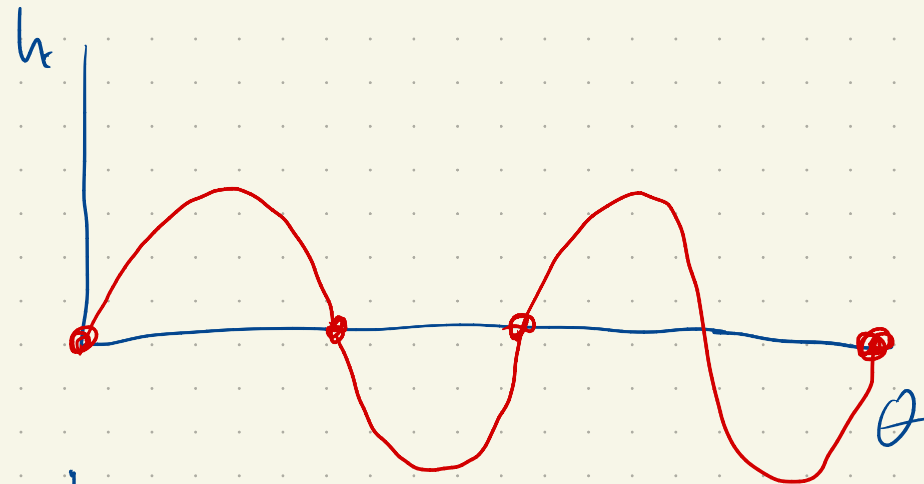
$$= -r^2\sin^2\theta + r^2\cos^2\theta$$

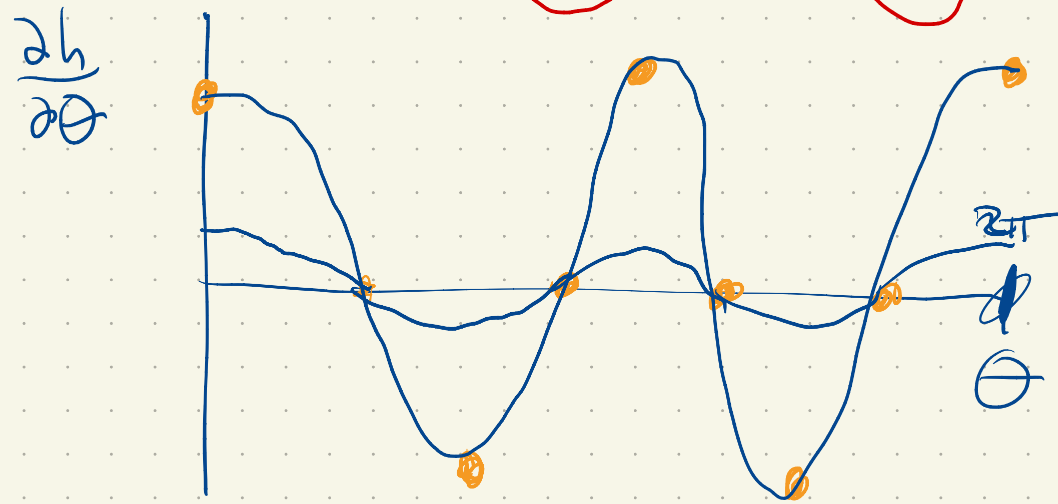
$$= r^2(\cos^2\theta - \sin^2\theta)$$

$$= r^2\cos(2\theta)$$

$h(x,y) = x \cdot y$







$$w = f\big(x(s,t,u),\ y(s,t,u),\ z(s,t,u)\big)$$

$$\frac{\partial w}{\partial s} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial s} + \frac{\partial f}{\partial z}\cdot\frac{\partial z}{\partial s}$$

$$\frac{\partial w}{\partial t}$$

$$\frac{\partial w}{\partial u}$$