$$z = f(x_0, y) \qquad \frac{\partial f}{\partial y}(x_0, y_0)$$

Equation of tangent plane

$$z = f(x_0, y_0) + \frac{\partial f}{\partial x}(x_0, y_0)(x - x_0) + \frac{\partial f}{\partial y}(x_0, y_0)(y - y_0)$$

$$f(x,y) = 2x^2 + y^2$$

$$(x_0, y_0) = (1,1)$$

$$f(1,1) = 3$$

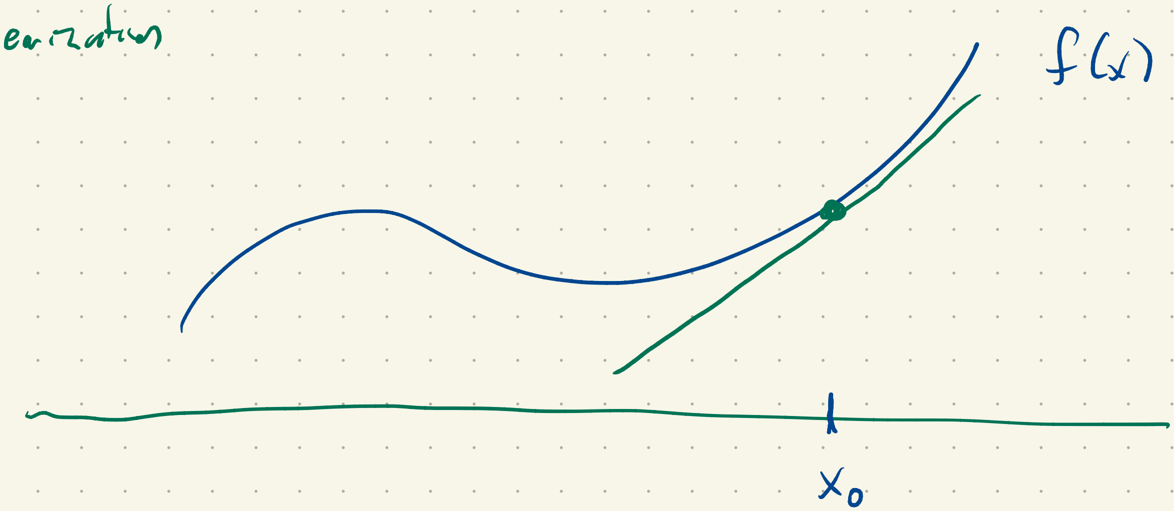
$$\frac{\partial f}{\partial x} = 4x \qquad \frac{\partial f}{\partial y} = 2y$$

$$\frac{\partial f}{\partial x}(1,1) = 4 \qquad \frac{\partial f}{\partial y}(1,1) = 2$$

$$z = 3 + 4(x-1) + 2(y-1)$$

$$4(x-1) + 2(y-1) - (z-3) = 0$$

Linearization



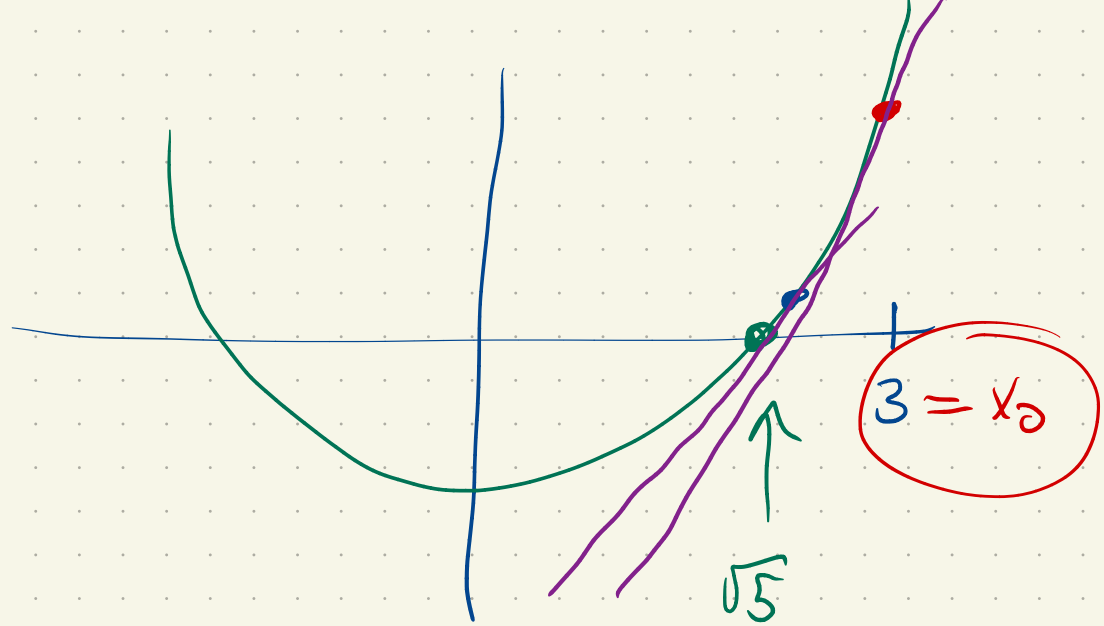
$$L(x) = f(x_0) + f'(x_0)(x - x_0)$$

$$3 + 7(x - 2)$$

$L(x) \approx f(x)$ for x near x_0 but $L(x)$ is easier to work with.

$$3 + 7(x-2) = 0$$

$F(x) = x^2 - 5$



$F(x_0)$

$F'(x_0)$

$F'(x) = 2x$

$F'(x_0) = 2x_0$

$$L(x) = F(x_0) + F'(x_0) \cdot (x - x_0)$$
$$= (x_0^2 - 5) + (2x_0)(x - x_0)$$

$$L(x) = 0$$

$$x - x_0 = \frac{5 - x_0^2}{2x_0}$$

$$x = x_0 + \frac{5 - x_0^2}{2x_0}$$

$$= x_0 + \frac{5}{2}\frac{1}{x_0} - \frac{x_0}{2}$$

$$= \frac{1}{2}x_0 + \frac{5}{2}\frac{1}{x_0}$$

$$x_0 = 3 \qquad = \frac{1}{2}\left(x_0 + \frac{5}{x_0}\right)$$

$$x_1 = \frac{1}{2}\left(x_0 + \frac{5}{x_0}\right) \qquad \sqrt{5} = 2.2360679774997 9$$

$$x_2 = \frac{1}{2}\left(x_1 + \frac{5}{x_1}\right) \qquad x_1 = \frac{1}{2}\left(3 + \frac{5}{3}\right)$$

$$x_3 = \frac{1}{2}\left(x_2 + \frac{5}{x_2}\right) \qquad x_1 = 2.333\overline{}$$

$$x_2 = \frac{1}{2}\left(x_1 + \frac{5}{x_1}\right)$$

$$x_2 = 2.2380952381$$

$$x_3 = 2.2360688564$$

$$x_4 = 2.2360679775$$

$f(x,y)$

Linearization at (x_0, y_0)

$$L(x,y) = f(x_0,y_0) + \frac{\partial f}{\partial x}(x_0,y_0)(x-x_0) + \frac{\partial f}{\partial y}(x_0,y_0)(y-y_0)$$

$\hookrightarrow$ graph of L is exactly the tangent plane to the graph of $f(x,y)$ at (x_0, y_0)

$L(x,y) \approx f(x,y)$ for (x,y) near (x_0,y_0)

but $L(x,y)$ is easier to work with.

$$P = \frac{8.2T}{V}$$ (units for P are kPa)
(1 mole of gas)

Linearization at $T = 300\,K$
$V = 30\,\ell$

$$P(300, 30) = 82 \text{ kPa}$$

$$\frac{\partial P}{\partial T} = \frac{8.2}{V} \qquad \frac{\partial P}{\partial T}(300, 30) = \frac{8.2}{30} = 0.273 \text{ kPa}/K$$

$$\frac{\partial P}{\partial V} = -\frac{8.2T}{V^2} \qquad \frac{\partial P}{\partial V}(300, 30) = -\frac{8.2 \cdot 300}{30^2} = -2.73 \text{ kPa}/\ell$$

$$L(T, V) = 82 + 0.273(T - 300) - 2.73(V - 30)$$

$P(T, V) \approx L(T, V)$ for (T, V) near $(300, 30)$

$$L(T,V) - 82 = 0.273(T-300) - 2.73(V-30)$$

$$dP = 0.273\, dT - 2.73\, dV$$

$T = 300K$

$V = 30\ell$

estimate change in pressure if we increase temp by 5 K and increase volume by 2 ℓ

+2

$$dP = 0.273 \cdot 5 - 2.73 \cdot 2$$

$$= -9.45 \text{ kPa}$$