Pure magic:

$$f(x,y) = x^3y^2 - x\ln(y)$$

$$f_x = 3x^2y^2 - \ln(y)$$

$$f_y = 2x^3y - \frac{x}{y}$$

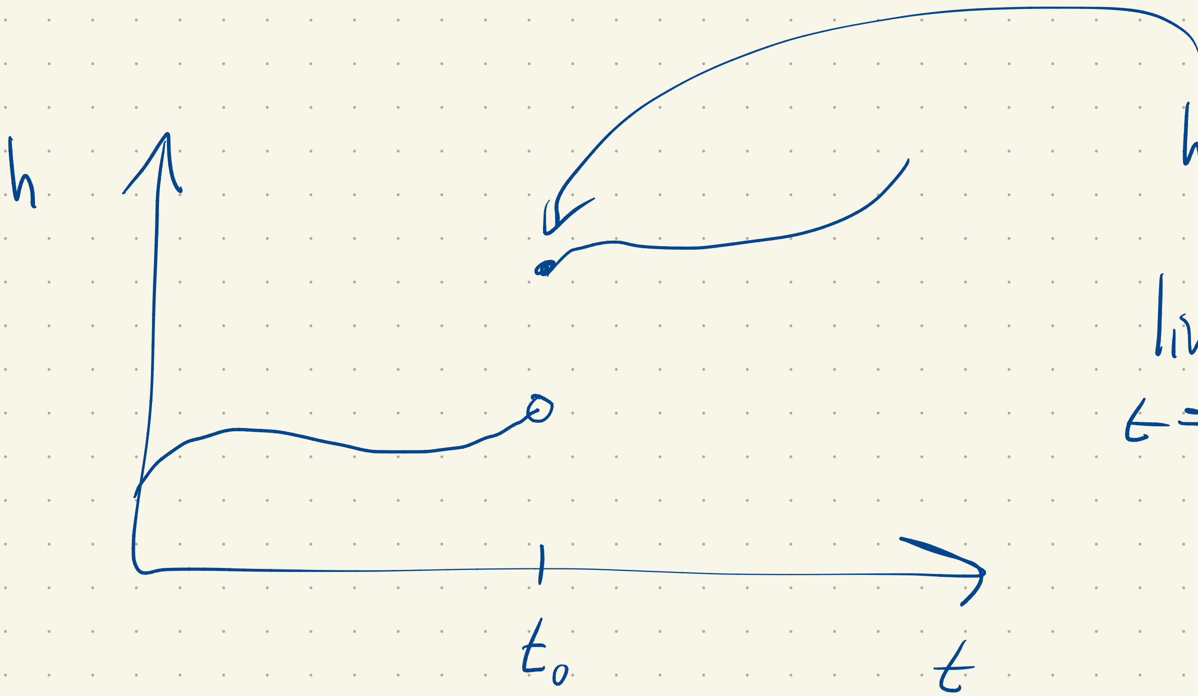
$$f_{xy} = 6x^2y - \frac{1}{y}$$

$$f_{xy} = f_{yx}$$

$$f_{yx} = 6x^2y - \frac{1}{y}$$

Up to legalese $f_{xy} = f_{yx}$ always.

$$f_{xxy} = f_{xyx} = f_{yxx}$$

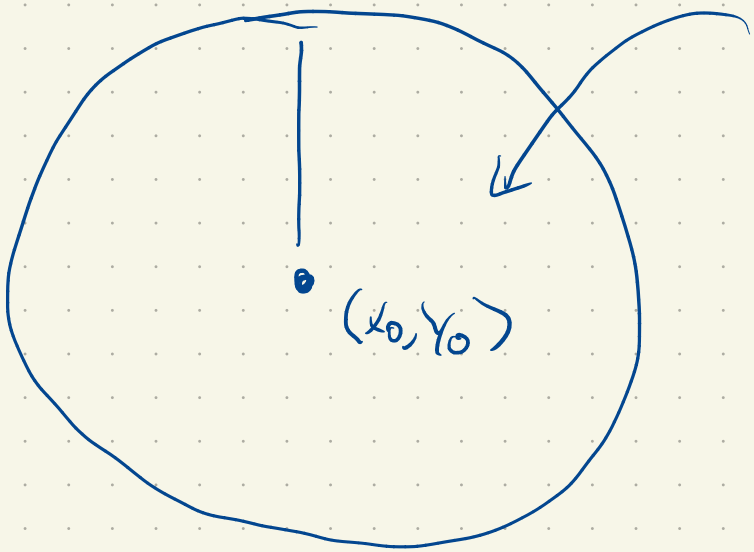


$h(t_0)$

$$\lim_{t \to t_0} h(t) = h(t_0)$$

$f(x,y)$ is continuous at (x_0, y_0) if

$$\lim_{(x,y) \to (x_0,y_0)} f(x,y) = f(x_0, y_0)$$



f_{xy}, f_{yx} need to exist and be continuous

$$\Rightarrow \quad f_{xy}(x_0, y_0) = f_{yx}(x_0, y_0)$$

Examples of continuous functions

- constants
- x, y
- sums, differences, products of continuous functions
- quotients of cts function (division by 0 is illegal)
- old friends: sin, cos, exp, ln ...
- compositions of cts function

Maxwell's Equation

electric field $E_1 \quad E_2 \quad E_3$

magnetic field $B_1 \quad B_2 \quad B_3$

charge density (C/m^3)

$$\left\{\begin{aligned} &\frac{\partial E_1}{\partial x} + \frac{\partial E_2}{\partial y} + \frac{\partial E_3}{\partial z} = \frac{1}{\varepsilon_0} \cdot \rho \\ &\frac{\partial B_1}{\partial x} + \frac{\partial B_2}{\partial y} + \frac{\partial B_3}{\partial z} = 0 \end{aligned}\right.$$

→ partial differential equations

- seismic vibrations
- ice sheet flow
- current flow (atmosphere, oceans)

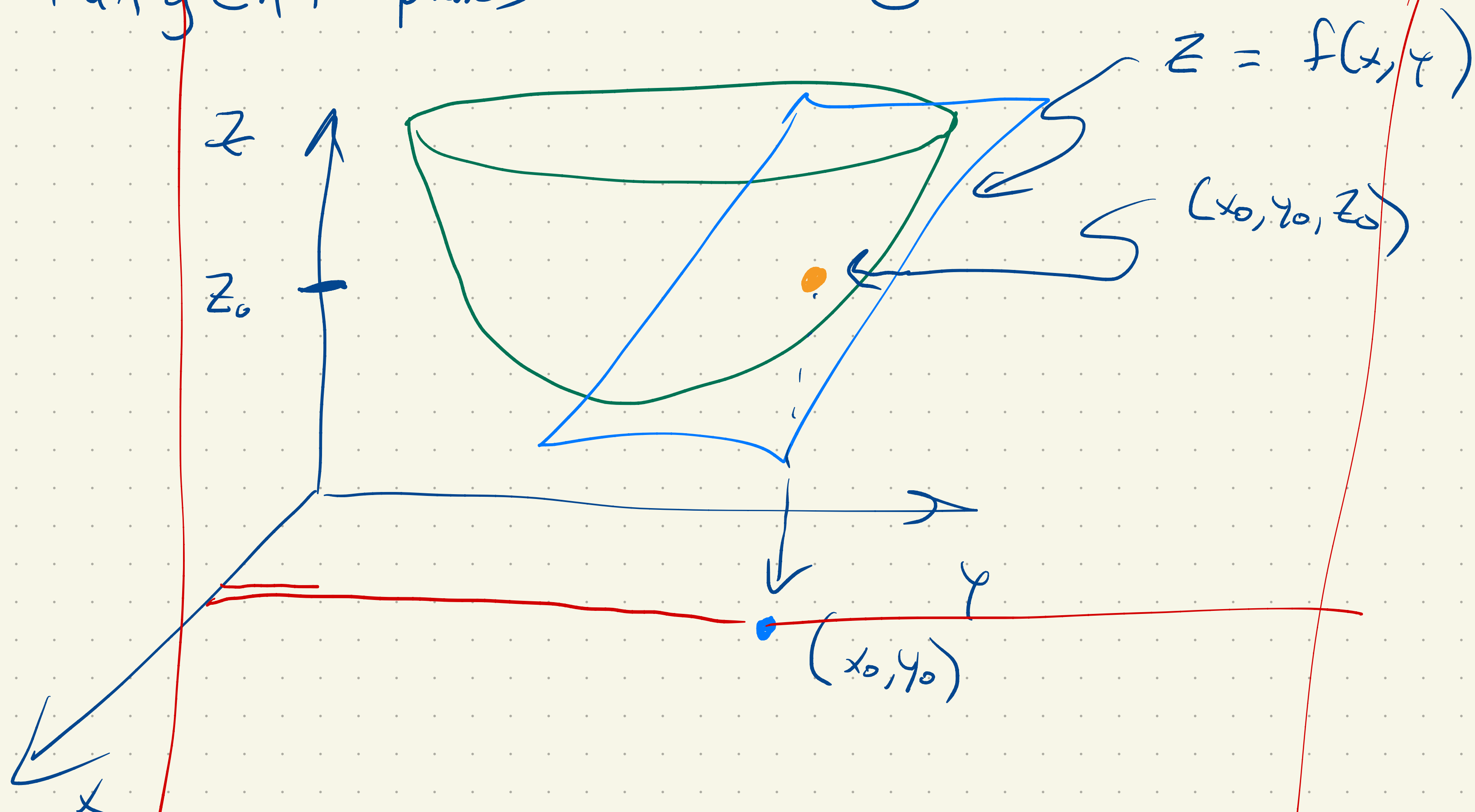
- E&M
- gravity (GR)
- heat transfer

$u(x,y,t)$

$$u_{tt} = c^2 (u_{xx} + u_{yy})$$

$\hookrightarrow c \to$ wave speed of the material $\left(\frac{\text{length}}{\text{time}}\right)$

Tangent planes to surfaces



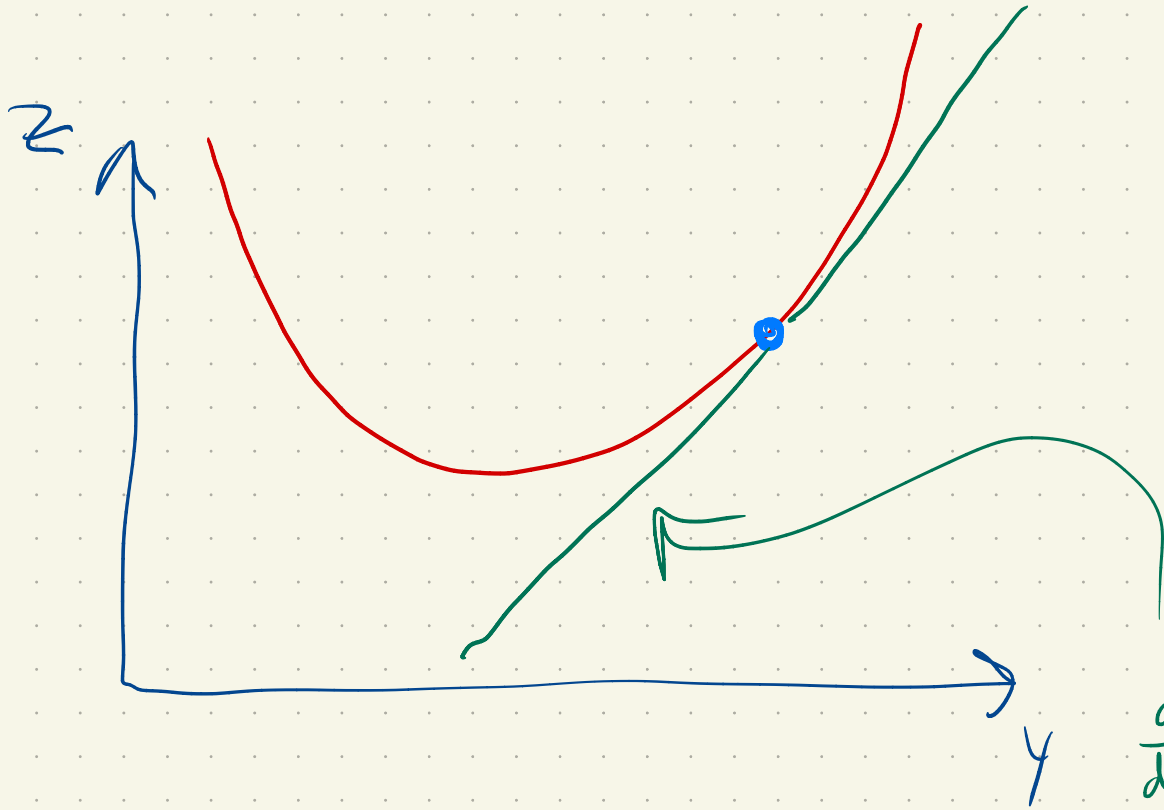
$$A(x-x_0) + B(y-y_0) + C(z-z_0) = 0$$

$$z = z_0 - \frac{A}{C}(x-x_0) - \frac{B}{C}(y-y_0)$$

$$z = z_0 + a(x - x_0) + b(y - y_0)$$

$$z_0 = f(x_0, y_0)$$

Let's look at just the plane $x = x_0$



slope is $\left.\frac{d}{dy}\right|_{y=y_0} f(x_0, y)$

$$z = f(x_0, y) \qquad \frac{\partial f}{\partial y}(x_0, y_0)$$

Equation of tangent plane

$$z = f(x_0, y_0) + \frac{\partial f}{\partial x}(x_0, y_0)(x - x_0) + \frac{\partial f}{\partial y}(x_0, y_0)(y - y_0)$$

$$f(x,y) = 2x^2 + y^2$$

$$(x_0, y_0) = (1,1)$$

$$f(1,1) = 3$$

$$\frac{\partial f}{\partial x} = 4x \qquad \frac{\partial f}{\partial y} = 2y$$

$$\frac{\partial f}{\partial x}(1,1) = 4 \qquad \frac{\partial f}{\partial y}(1,1) = 2$$

$$z = 3 + 4(x-1) + 2(y-1)$$

$$4(x-1) + 2(y-1) - (z-3) = 0$$