4.3? Partial Derivatives

$$f(x,y) = e^x \cos(x^3y^2)$$

$$\frac{\partial f}{\partial x} = e^x \cos(x^3y^2) - e^x \sin(x^3y^2) \cdot (3x^2y^2)$$

$\uparrow$

partial derivative
of f with respect to x

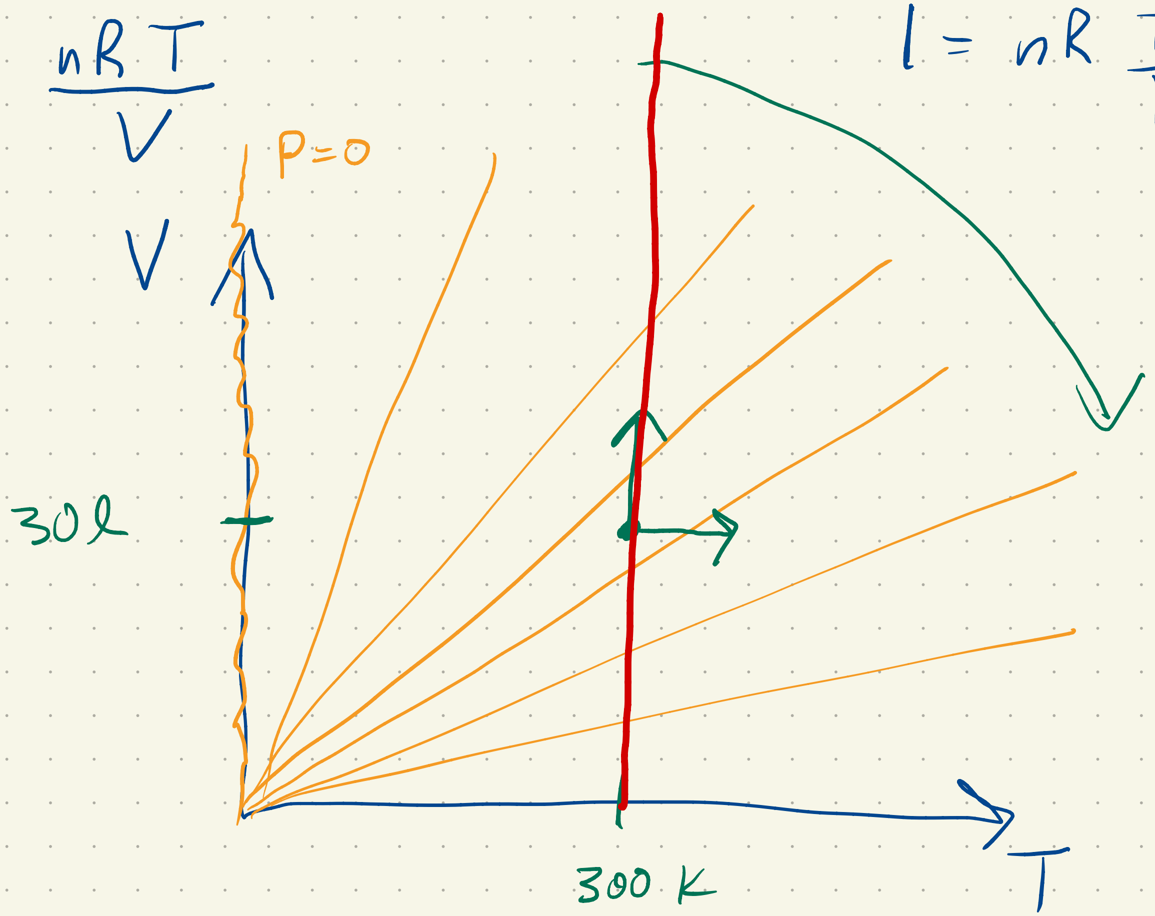
$$\frac{d}{dy} e^7 \cos(7^3y^2) = -e^7 \sin(7^3y^2) \cdot (2 \cdot 7^3 y)$$

$$\frac{\partial}{\partial y} e^x \cos(x^3y^2) = -e^x \sin(x^3y^2) \cdot (2x^3y)$$

$$P = \frac{nRT}{V}$$

$$1 = nR\,\frac{T}{V}$$

$$V = (nR)\,T$$



increasing pressure

1 mole of gas

$$R = 0.082 \ \frac{\text{atm } l}{\text{mol K}}$$

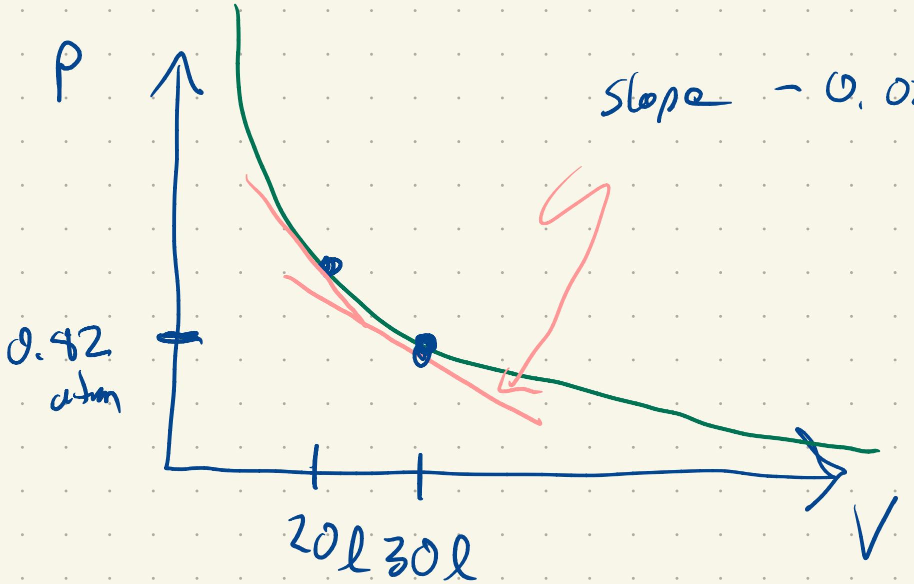
$$P = 0.82 \text{ atm}$$

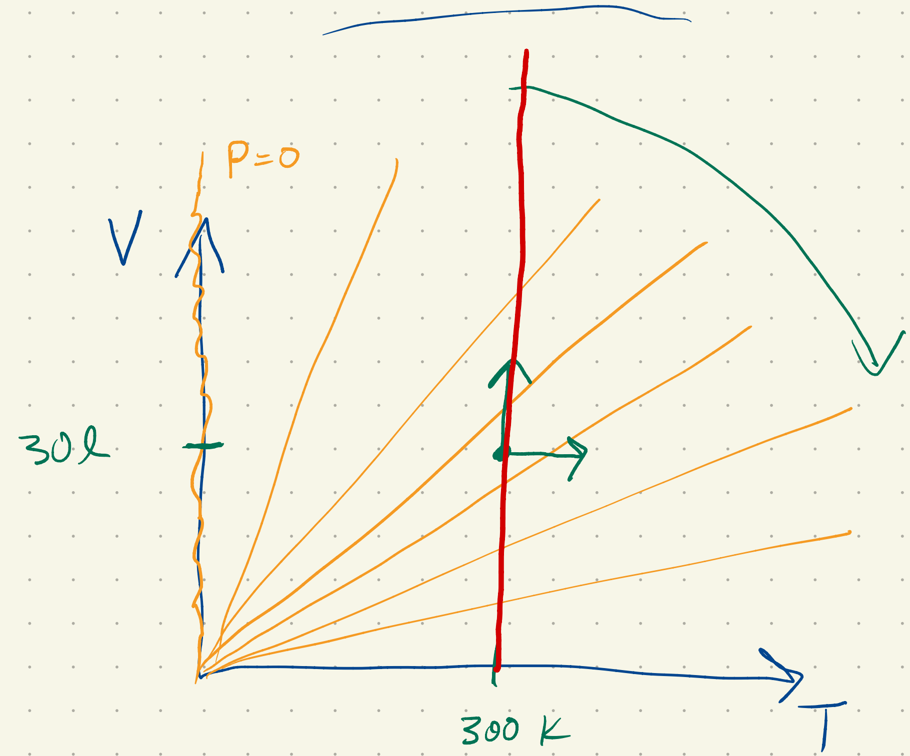
$$P = \frac{0.082 \cdot 300}{V}$$

$$\frac{dP}{dV} = -\frac{0.082 \cdot 300}{V^2} \qquad \frac{atm}{l}$$

$$\left.\frac{dP}{dV}\right|_{V=30l} = -0.027 \ \frac{atm}{l}$$

slope -0.027





$$P = \frac{0.082 \cdot 300}{V}$$

$$\left.\frac{dP}{dV}\right|_{V=30\,l} = -0.027 \ \frac{atm}{l}$$

$$\frac{\partial P}{\partial V} = \boxed{-\frac{0.082 \cdot T}{V^2}}$$

$$\left(P = \frac{0.082\,T}{V} \right)$$

If the temp of the gas is T and the volume is V and we change the volume the pressure changes at a rate of

$$-\frac{0.082 \cdot T}{V^2}.$$

$T = 300,\ V = 20 \Rightarrow \frac{\partial P}{\partial V} = -0.0615$ atm / l

$T = 300,\ V = 30 \Rightarrow \frac{\partial P}{\partial V} = -0.027$ atm / l

Ballpark: how much should I increase the volume from 30l to decrease the pressure by 0.1 atm

(0.82 atm at T=300K, V=30l)

$$-0.027 \frac{atm}{l} \cdot \Delta V l = -0.1 \ atm$$

$$\Delta V = \frac{-0.1}{-0.027} = 3.7 l$$

$$P = \frac{0.082 \cdot 300}{33.7} = 0.73 \ atm$$

Summary: Give $f(x,y)$

$\frac{\partial f}{\partial x}(x_0, y_0)$ is the rate of change of f with respect to x at $x = x_0$, $y = y_0$.

Second partial derivatives

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\frac{\partial}{\partial x} f = f_{xx}$$

$$\frac{\partial^2 f}{\partial y\, \partial x} = \frac{\partial}{\partial y}\frac{\partial}{\partial x} f = f_{xy} \qquad f_{xy}$$

$$\frac{\partial^2 f}{\partial x\, \partial y} = \frac{\partial}{\partial x}\frac{\partial}{\partial y} f = f_{yx}$$

Pure magic:

$$f(x,y) = x^3y^2 - x\ln(y)$$

$$f_x = 3x^2y^2 - \ln(y)$$

$$f_y = 2x^3y - \frac{x}{y}$$

$$f_{xy} = 6x^2y - \frac{1}{y}$$

$$f_{yx} = 6x^2y - \frac{1}{y}$$

Up to legalese $f_{xy} = f_{yx}$ always.

$$f_{xxy} = f_{xyx} = f_{yxx}$$