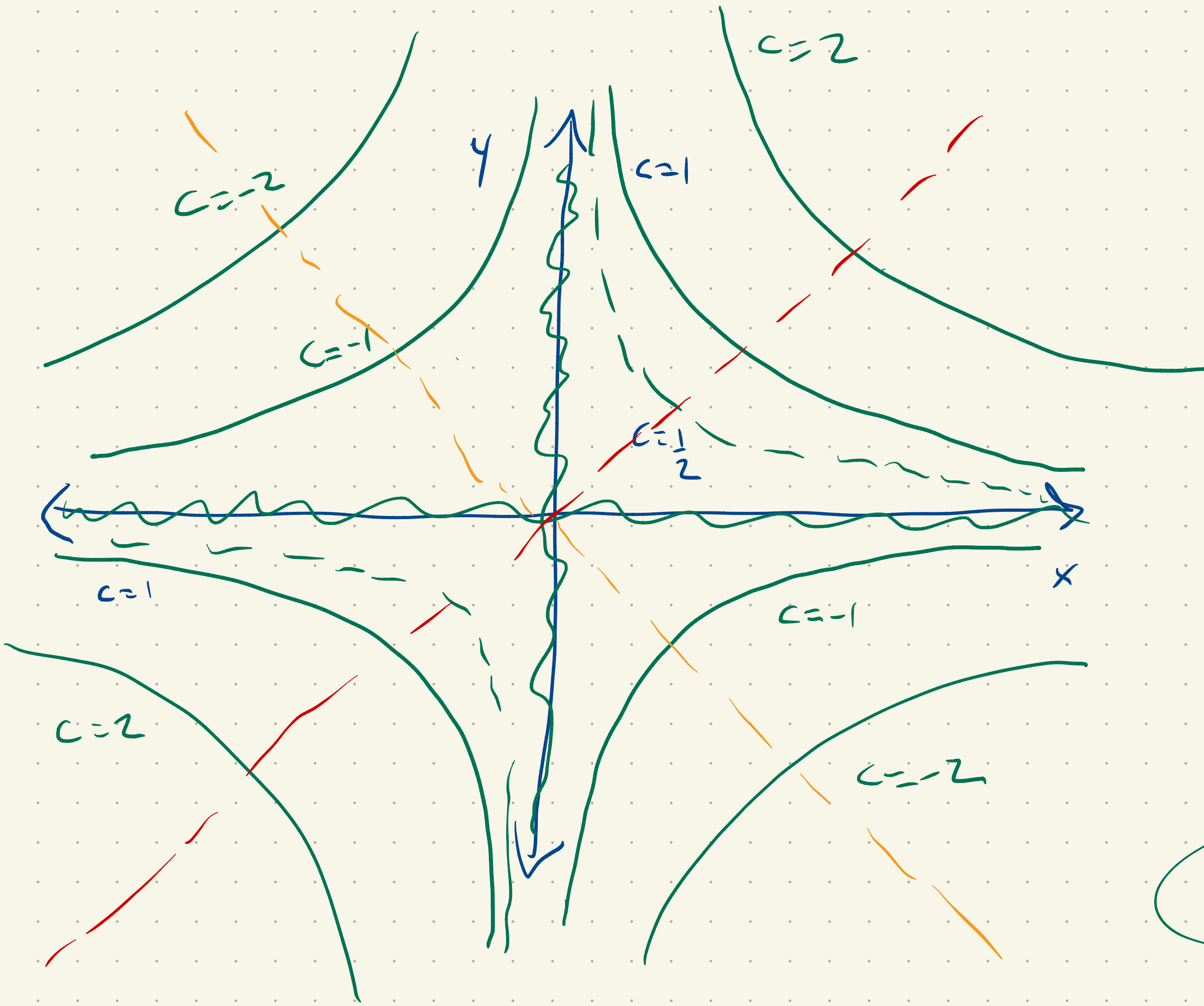
$$f(x,y) = xy$$

level set for the value c is the set of points (x,y) where $f(x,y) = c$.

$x^2 - y^2$



$c = 0$

$xy = 0$

$c = 1$

$xy = 1 \Rightarrow y = \frac{1}{x}$

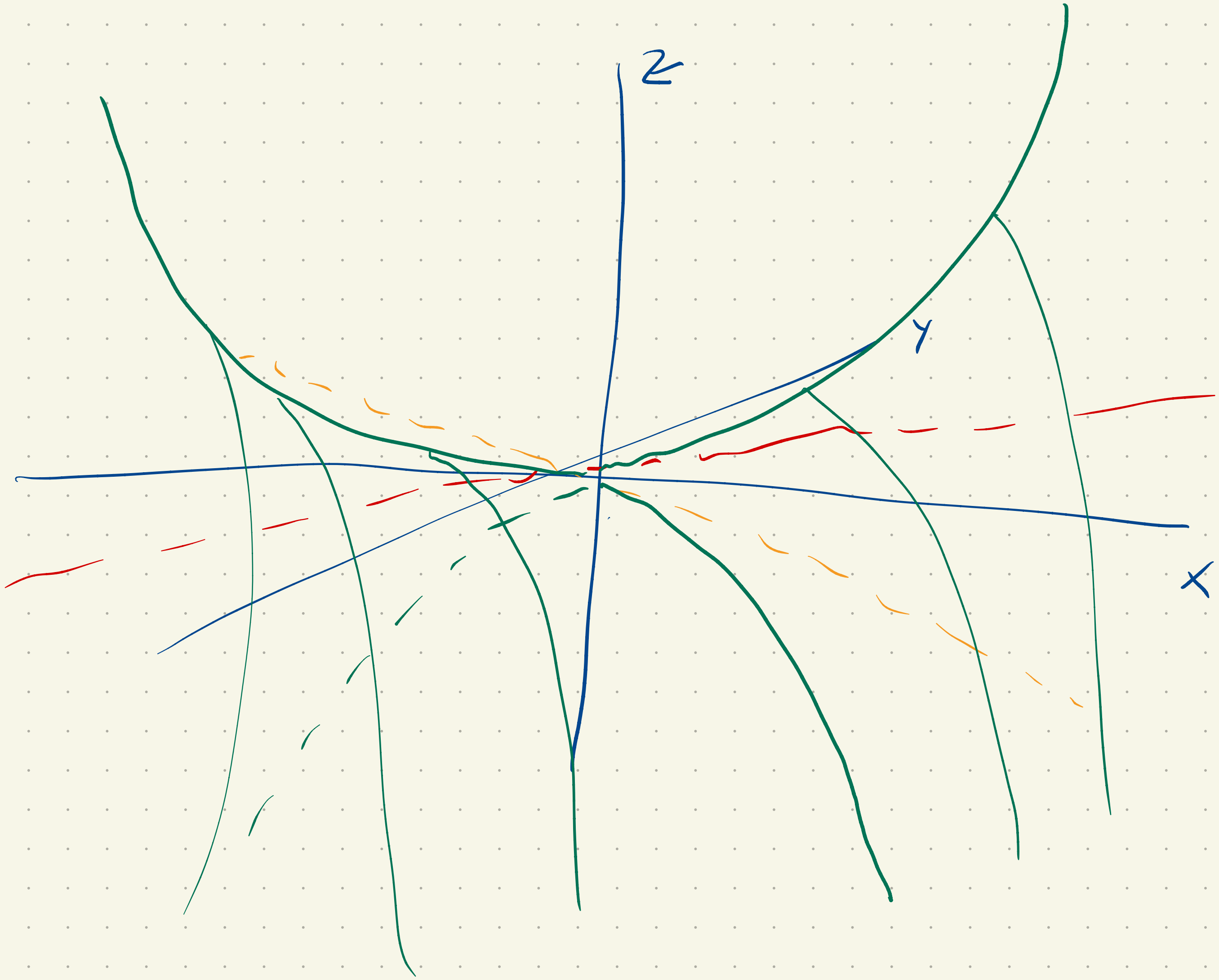
$c = \frac{1}{2}$

$xy = \frac{1}{2} \Rightarrow y = \frac{1}{2}\frac{1}{x}$

$c = -1 \Rightarrow xy = -1$

$y = \frac{-1}{x}$

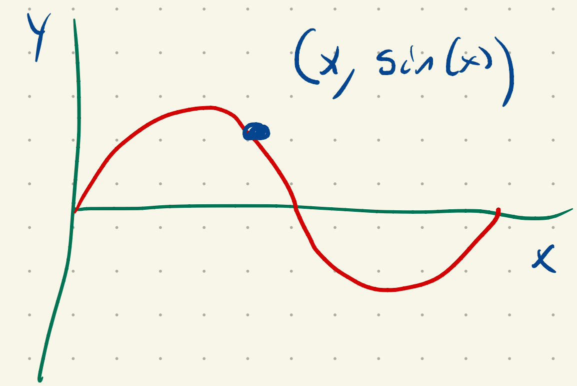
$c = -2$ $\quad xy = -2$



$$f(x,y) = xy$$

$$(x, y, f(x,y))$$

$$z = f(x,y)$$



4.2 Limits and Continuity

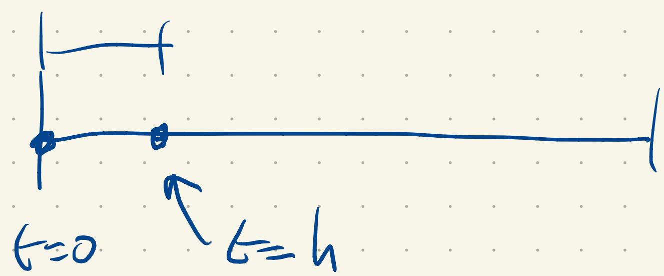
Why do we care about limits? $\frac{0}{0}$

$T(t)$

temp (°F) time (minutes)

$T(0) = 67$ °F

$T(60) = 61$ °F



change in temp after the hour

$$\frac{T(60) - T(0)}{60} = \frac{-6}{60} = -0.1 \text{ °F/min}$$

$$T(1) = 66.8 \text{ °F}$$

$$\frac{T(1) - T(0)}{1} = \frac{66.8 - 67}{1} = -0.2 \text{ °F/min}$$

$$\frac{T(h) - T(0)}{h}$$

$$\frac{T(0) - T(0)}{0} = \frac{67 - 67}{0} = \frac{0}{0}$$

$$\frac{\sin(x)}{x} \leftarrow$$

$$\frac{\sin(0)}{0} = \text{err}$$

$$\frac{\sin(1)}{1} =$$

$$\frac{\sin(0.1)}{0.1} = 0.0174524064$$

$$\frac{2\pi}{360}$$

$$\frac{\sin(0.01)}{0.01} = 0.01745329924$$

$$0.01745329925$$

$$\frac{\sin(0.001)}{0.001} = 0.01745329925$$

$$\lim_{x \to 0} \frac{\sin d(x)}{x} = \frac{2\pi}{360}$$

$$\lim_{x \to 0} \frac{\sin(x)}{x} = 1$$

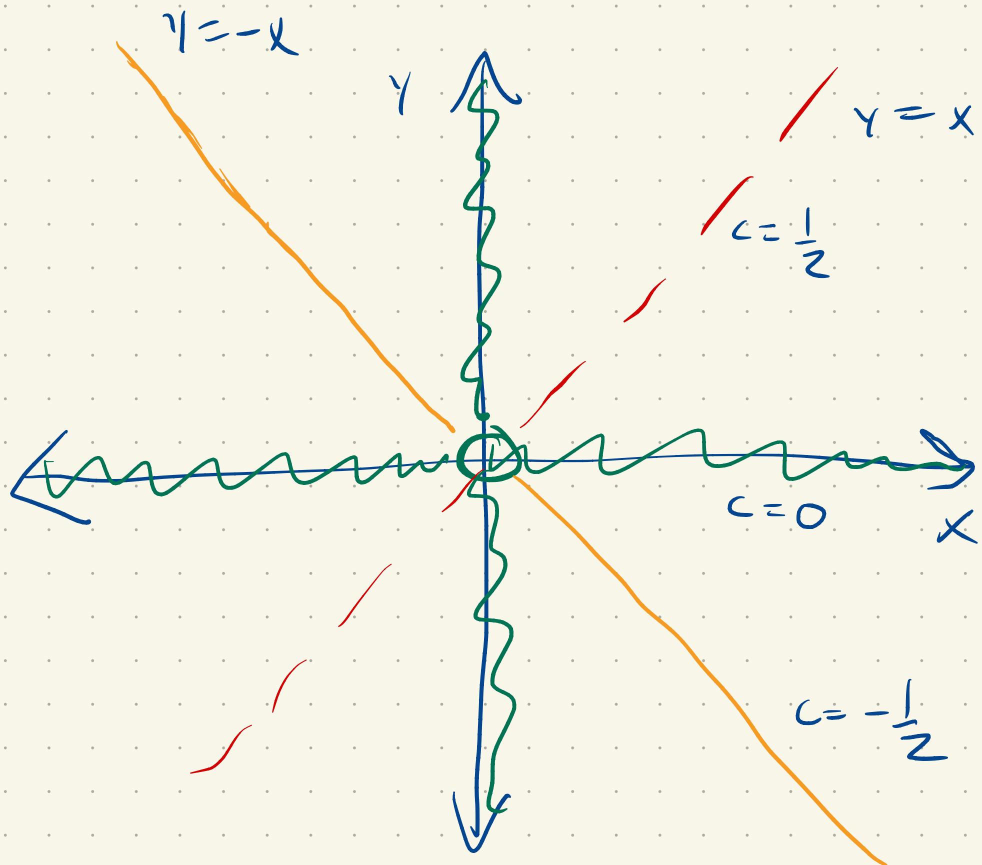
$$f(x,y) = \frac{xy}{x^2+y^2}$$

$f(0,0) = \frac{0}{0}$ uh oh!

$$\lim_{(x,y)\to(0,0)} f(x,y)?$$

If $y = x$

$$f(x,x) = \frac{x \cdot x}{x^2+y^2} = \frac{x^2}{2x^2} = \frac{1}{2}$$



If $y = -x$

$$f(x,y) = f(x,-x) = \frac{x \cdot (-x)}{x^2 + (-x)^2} = \frac{-x^2}{2x^2} = -\frac{1}{2}$$

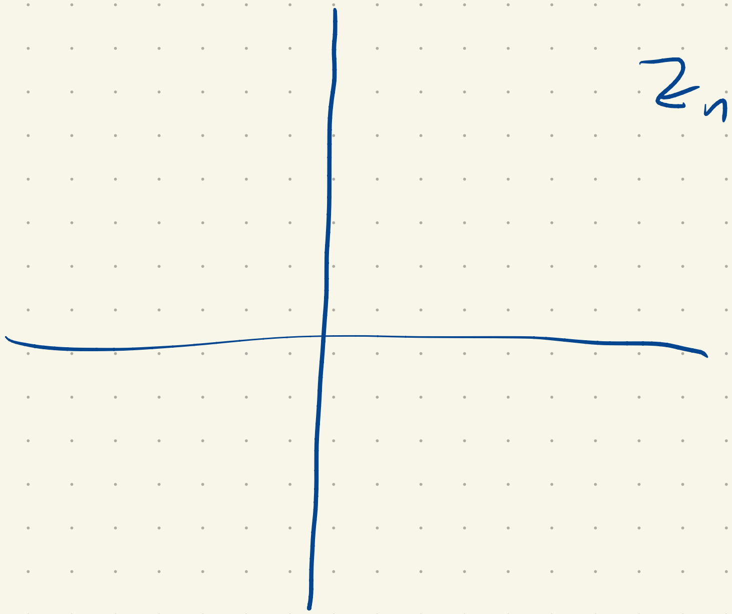
$$\boxed{\lim_{(x,y)\to(0,0)} f(x,y) \text{ does not exist}}$$

$\lim_{(x,y)\to(0,0)} f(x,y) = L$ requires the following:

given any sequence of points (x_n, y_n) with

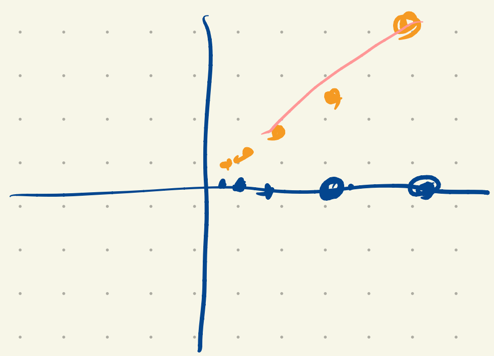
$$x_n \to 0$$
$$y_n \to 0$$

The function values $z_n = f(x_n, y_n)$ satisfy



$z_n \longrightarrow L$ (no matter what sequence of (x_n, y_n).

The $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy}{x^2+y^2}$ does not exist:



$x_n = \frac{1}{n}$, $y_n = 0$

$z_n = f(x_n, y_n) = 0$

$z_n \longrightarrow \boxed{0}$

$\hat{x}_n = \frac{1}{n}$ $\hat{y}_n = \frac{1}{n}$

$\hat{z}_n = f(\hat{x}_n, \hat{y}_n) = \frac{1}{2}$

$$\hat{z}_1 \longrightarrow \boxed{\frac{1}{2}}$$

$\boxed{0} \neq \boxed{\frac{1}{2}}$ so

the limit doesn't exist

$$f(x,y) = \frac{xy}{\sqrt{x^2+y^2}}$$

$f(0,0) = \frac{0}{0}$ uh oh

claim $\lim_{(x,y)\to(0,0)} f(x,y) = 0$

$x = r\cos\theta$

$y = r\sin\theta$

$$\frac{r\cos\theta \cdot r\sin\theta}{\sqrt{r^2\cos^2\theta + r^2\sin\theta}} = \frac{r^2\cos\theta\sin\theta}{r}$$

$$= r\cos\theta\sin\theta$$

$$= \frac{r}{2} \sin(2\theta)$$

$$-\frac{r}{2} \leq f(x,y) \leq \frac{r}{2}$$

$$-\frac{\sqrt{x^2+y^2}}{2} \leq f(x,y) \leq \frac{\sqrt{x^2+y^2}}{2}$$