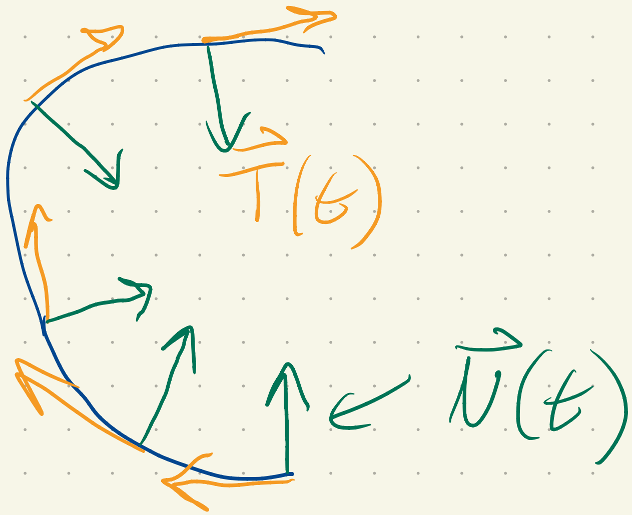


$$\frac{d}{dt}\vec{T}\cdot\vec{T} = 0$$

$$\vec{T}\cdot\vec{T}' + \vec{T}'\cdot\vec{T} = 0$$

$$\vec{T}\cdot\vec{T}' = 0$$

how much turning

$\vec{r}(t)$

$$\frac{\vec{r}\,'(t)}{\|\vec{r}\,'(t)\|}$$

$$\vec{N} = \frac{\vec{T}'}{\|\vec{T}'\|}$$

normal vector: points in the direction of turning

$\vec{r}(t)$ $\qquad$ $\vec{T}(t)$

$\vec{r}\,'(t)/\|\vec{r}\,'(t)\|$

unit length

$$\vec{a}(t) = a_T \vec{T} + a_N \vec{N}$$

tangential component of acceleration (speedup)

normal component of acceleration (turning)

Task: compute the tangential or normal components of acceleration

$$\vec{a} = a_T \vec{T} + a_N \vec{N}$$

$$\vec{a} \cdot \vec{T} = (a_T \vec{T} + a_N \vec{N}) \cdot \vec{T}$$

$$= a_T \underbrace{\vec{T}\cdot\vec{T}}_{1} + a_N \underbrace{\vec{N}\cdot\vec{T}}_{0}$$

$$= a_T$$

$$\vec{a}\cdot\vec{N} = a_N$$

$$\vec{r}(t) = \langle \cos(t^2), \sin(t^2) \rangle \qquad \sqrt{4t^2} = 2|t|$$

$$\vec{r}\,'(t) = \langle -2t\sin(t^2), 2t\cos(t^2) \rangle = 2t\langle -\sin(t^2), \cos(t^2) \rangle$$

$$\vec{T}(t) = \frac{\vec{r}\,'(t)}{\|\vec{r}\,'(t)\|} = \langle -\sin(t^2), \cos(t^2) \rangle \qquad (t > 0$$

$$\langle \sin(t^2), -\cos(t^2) \rangle \qquad t < 0)$$

$$\vec{N} = \frac{T'(t)}{\|T'(t)\|}$$

$$\vec{T}'(t) = \langle -2t\cos(t^2), -2t\sin(t^2) \rangle$$

$$\|\vec{T}'(t)\| = 2t \qquad (t > 0)$$

$$\vec{N} = \langle -\cos(t^2), -\sin(t^2) \rangle$$

$$\vec{T} = \langle -\sin(t^2), \cos(t^2) \rangle$$

$$\vec{r}'(t) = \langle -2t\sin(t^2), 2t\cos(t^2) \rangle$$

$$\vec{a}(t) = \vec{r}''(t) =$$

$$\langle -2\sin(t^2) - 4t^2\cos(t^2),$$
$$2\cos(t^2) - 4t^2\sin(t^2) \rangle$$

tangential component of acceleration:

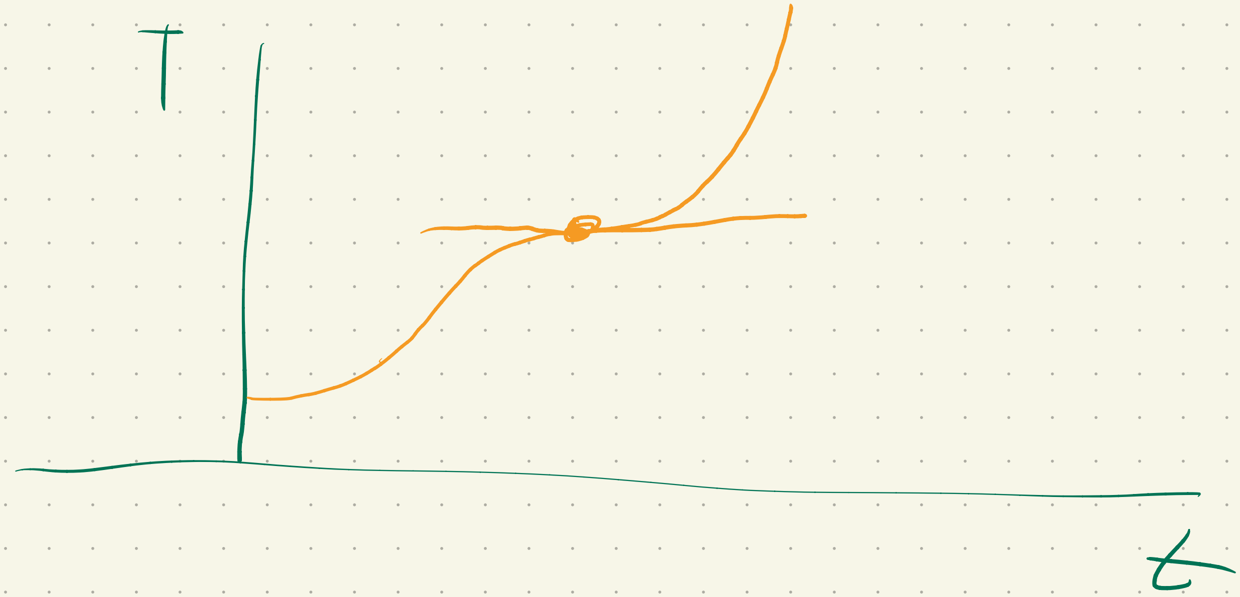
$$\vec{a} \cdot \vec{T} = \cdot \langle -\sin(t^2), \cos(t^2) \rangle$$

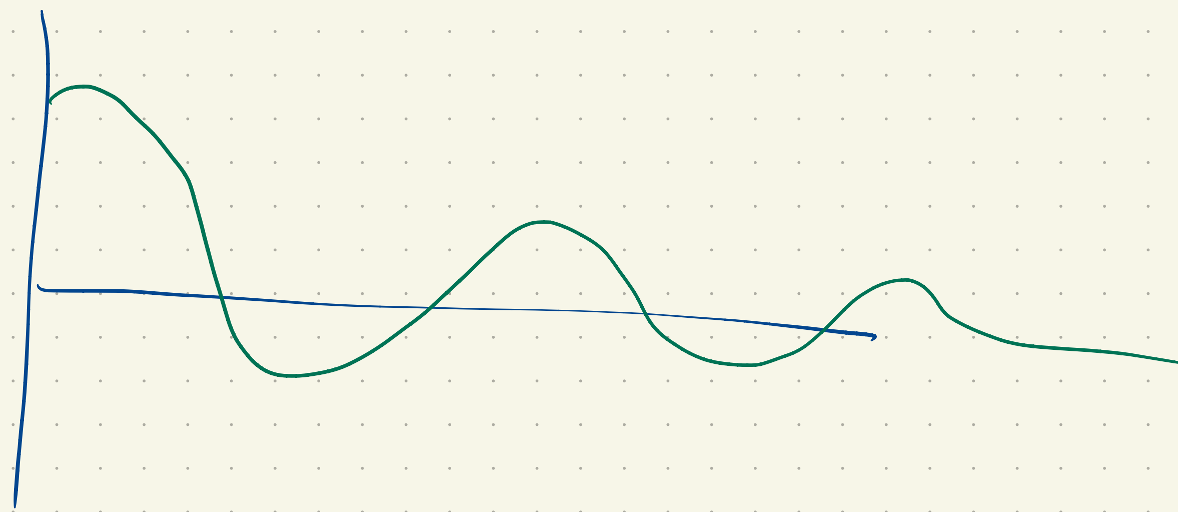
$$= 2$$

$$\vec{a} = 2\vec{T} + 4t^2\vec{N}$$

$$\vec{a} \cdot \vec{N} = 4t^2$$

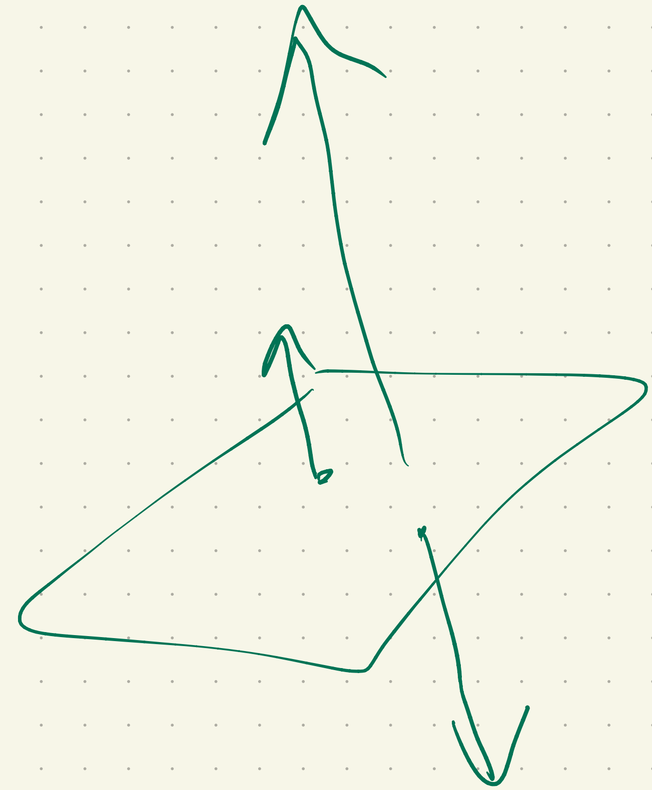
$T(t)$





$5x - 3y + 7z = 19$

$\langle 5, -3, 7 \rangle = \vec{n}$



$\langle 1, 0, 2 \rangle$ lies on the plane.

$5 \cdot 0 - 3 \cdot 0 + 7z = 19$

$z = \frac{19}{7}$

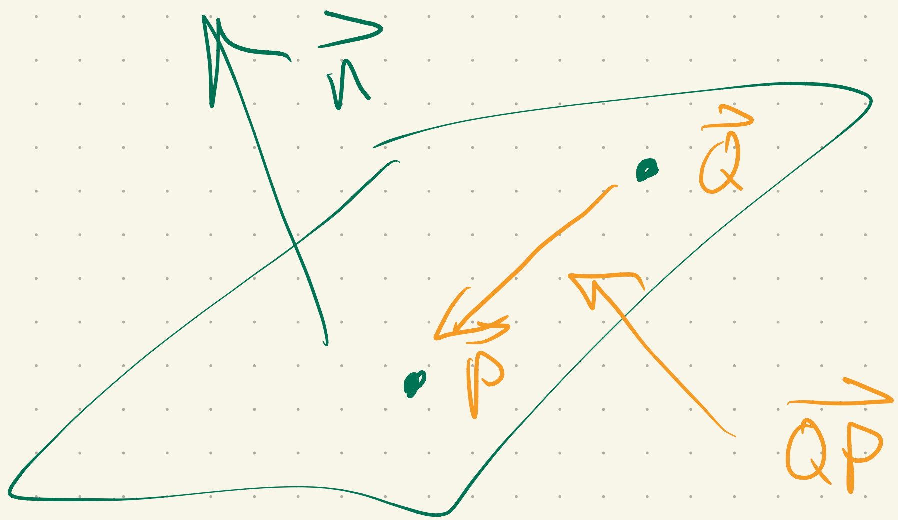
$\langle 0, 0, \frac{19}{7} \rangle$

$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

$$5(x-0)-3(y-0)+7\left(z-\frac{19}{7}\right)=0$$

$$5x-3y+7z=19$$

$$5(x-1)-3(y-0)+7(z-2)=0$$



$\vec{x}, \vec{y}$

$\vec{x} \times \vec{y} \perp \vec{x}, \vec{y}$

$$\underbrace{\langle 1, 0, 2\rangle}_{\vec{P}} - \underbrace{\langle 0, 0, \tfrac{19}{7}\rangle}_{\vec{Q}} = \underbrace{\langle 1, 0, -\tfrac{5}{7}\rangle}_{\overrightarrow{QP}}$$

$$\vec{n} = \langle 5, -3, 7\rangle$$

$$\vec{n} \cdot \overrightarrow{QP} = 5 \cdot 1 - 3 \cdot 0 - \frac{5}{7} \cdot 7 = 5 - 5 = 0 \ \checkmark$$

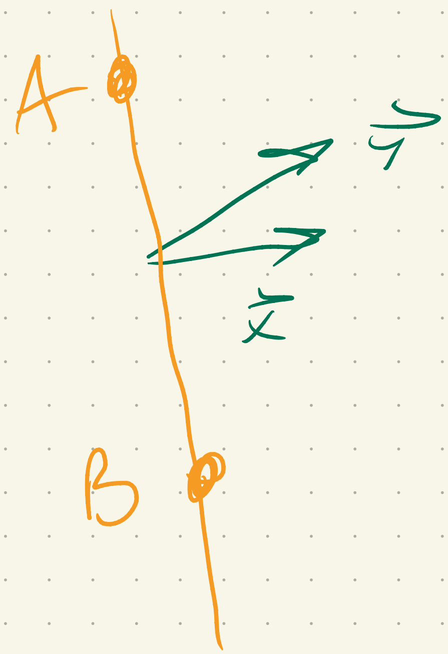
$$\vec{x} \cdot \vec{y} = \|\vec{x}\| \, \|\vec{y}\| \cos\theta$$

measure of similarity

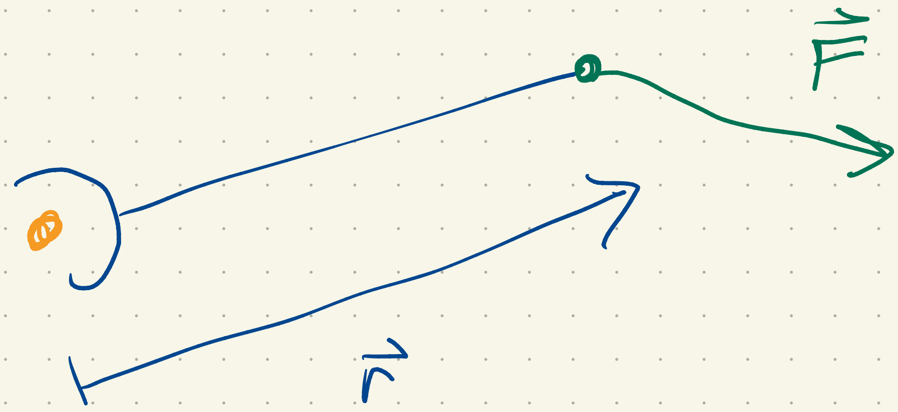
$$\vec{x} \cdot \vec{y} < 0$$

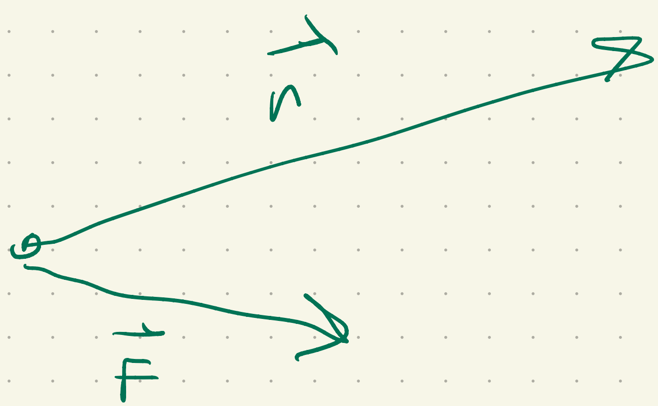
$\vec{x} \times \vec{y}$ is a vector

it's perpendicular to $\vec{x}$ and $\vec{y}$

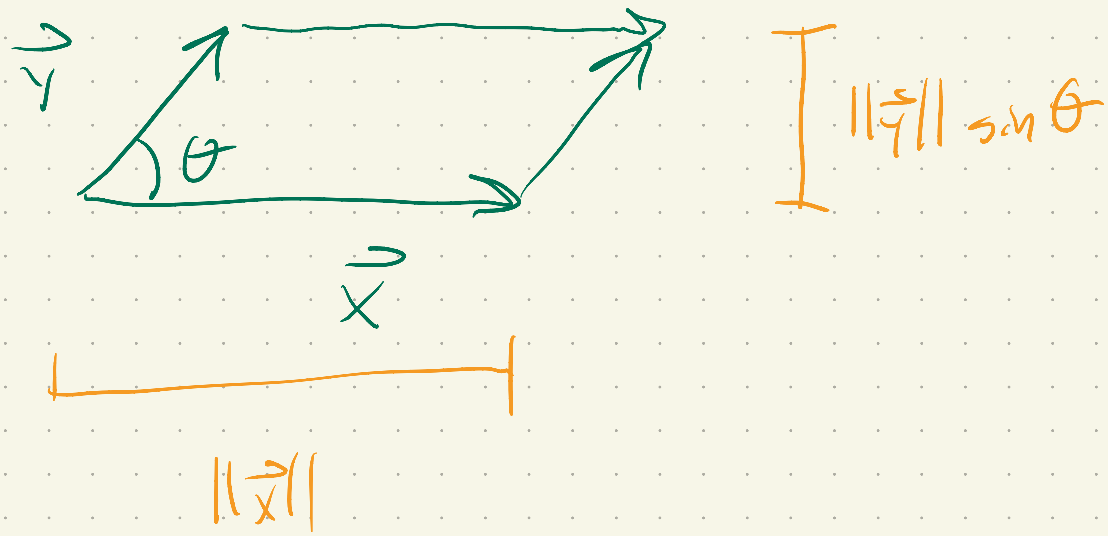


$$\vec{\tau} = \vec{r} \times \vec{F}$$





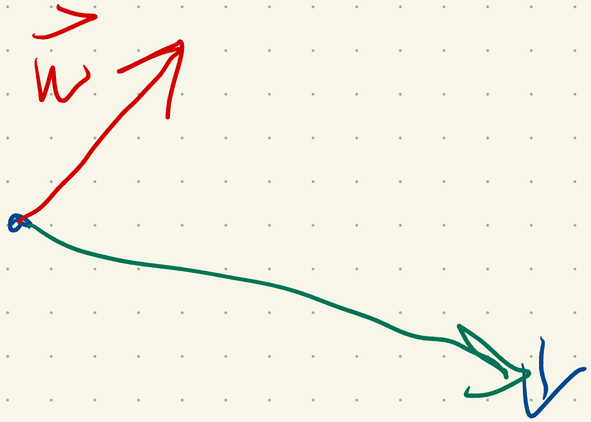
$$\|\vec{x} \times \vec{y}\| = \|\vec{x}\| \, \|\vec{y}\| \sin\theta$$



$$\vec{v} = 5\hat{i} - 2\hat{j}$$

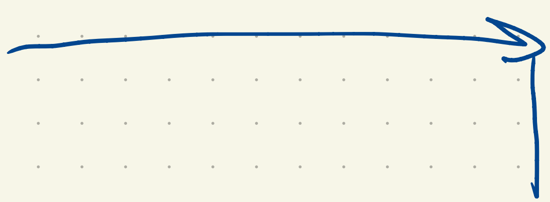
$$\vec{v} \cdot \hat{i} = 5$$

$$\vec{v} \cdot \hat{j} = -2$$



$$\vec{w} = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$$

$\vec{w} \cdot \vec{v}$ tells you how much $\vec{v}$ is pointing in the $\vec{w}$ direction



$$\vec{v} = c\vec{w} + \vec{s} \qquad \vec{s} \perp \vec{w}$$

$$\vec{v} \cdot \vec{w} = (c\vec{w} + \vec{s}) \cdot \vec{w}$$

$$= c \underbrace{\vec{w} \cdot \vec{w}}_{1} + \underbrace{\vec{s} \cdot \vec{w}}_{= 0}$$

$$= c$$

$\vec{v} \cdot \vec{w}$ tells you how much $\vec{v}$ is pointing in the $\vec{w}$ direction

$\downarrow$

$\vec{w}$ is a unit vector.

$$\text{proj}_{\vec{u}} \vec{v} = \frac{\vec{v} \cdot \vec{u}}{\|\vec{u}\|^2} \vec{u}$$

(any vector $\vec{u}$, not just unit

vectors)

$$= \vec{v} \cdot \left(\frac{\vec{u}}{\|\vec{u}\|} \right) \frac{\vec{u}}{\|\vec{u}\|}$$

$\vec{u} = \hat{\imath}$

$\vec{v} = 5\hat{\imath} - 2\hat{\jmath}$

$$\text{proj}_{\hat{\imath}}\, \vec{v} = \frac{\vec{v} \cdot \hat{\imath}}{\|\hat{\imath}\|} \frac{\hat{\imath}}{\|\hat{\imath}\|}$$

$$= \frac{5}{1} \frac{\hat{\imath}}{1}$$

$$= 5\hat{\imath}$$