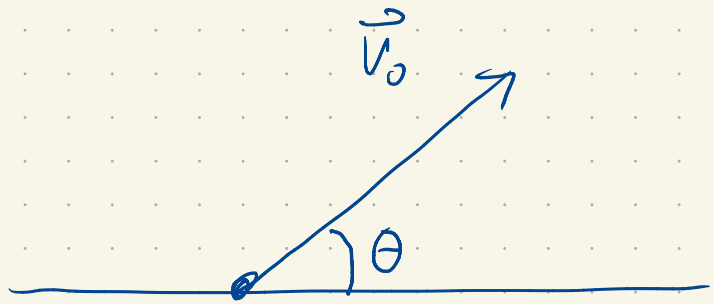
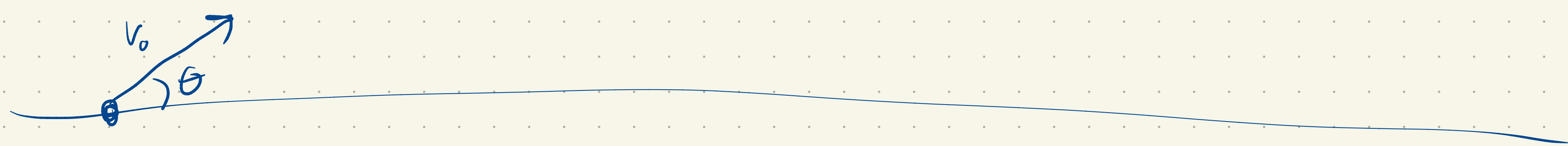
E.g. $\vec{r}_0 = 0$



$$\vec{v}_0 = v_0 \cos(\theta)\,\hat{i} + v_0 \sin(\theta)\,\hat{k}$$

$$\begin{aligned}
\|\vec{v}_0\|^2 &= v_0^2 \cos^2(\theta) + v_0^2 \sin^2(\theta) \\
&= v_0^2 (\cos^2\theta + \sin^2\theta) \\
&= v_0^2
\end{aligned}$$

$$\|\vec{v}_0\| = v_0$$



$\vec{r}_0 = \vec{0}$

Q's: 1) how far does the projectile travel?

2) how long to get there

3) maximum height?

(in terms of v_0, θ)

$$\vec{r}(t) = -\frac{9.8}{2}t^2\hat{k} + t\vec{v}_0 + \vec{r}_0$$

$$= -\frac{9.8}{2}t^2\hat{k} + t\left[v_0\cos\theta\,\hat{\imath} + v_0\sin\theta\,\hat{k}\right]$$

$$= \left[-\frac{9.8}{2}t^2 + t v_0\sin\theta\right]\hat{k} + t v_0\cos\theta\,\hat{\imath}$$

$$-\frac{9.8}{2}t^2 + t\,v_0 \sin\theta = 0$$

$$t\left(-\frac{9.8}{2}t + v_0 \sin\theta\right) = 0$$

either $t = 0$ or $-\frac{9.8}{2}t + v_0 \sin\theta = 0$

$$t = \frac{2v_0 \sin\theta}{9.8}$$

amount of time before returning to earth.

distance traveled $t \cdot \cos\theta\, v_0 = \frac{2\,v_0^2 \sin\theta\cos\theta}{9.8}$

$v_0 = 150$ m/s $\quad \theta = 45^\circ = \frac{\pi}{4}$

$$t = \frac{2 \cdot 150 \sin(\pi/4)}{9.8} \approx 21.64\ s$$

$$x = 21.64 \cdot \cos(\pi/4)\, 150 = 2295\, m$$

Max height?

$$z = -\frac{9.8}{2} t^2 + t v_0 \sin\theta$$

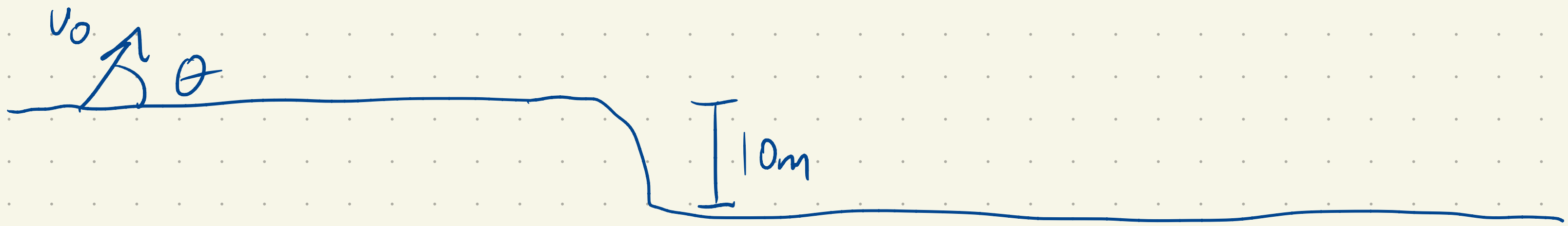
$$z' = -9.8t + v_0 \sin\theta$$

$z' = 0$ happens when $t = \dfrac{v_0 \sin\theta}{9.8}$

$v_0 = 150$ m/s, $\theta = 45°$

$$t = 10.82$$

$z(t)$ $\quad z(10.82) = 573\, m$



$$z = -\frac{9.8}{2}t^2 + t v_0 \sin\theta$$

$$-\frac{9.8}{2}t^2 + t v_0 \sin\theta = -10$$

$$-\frac{9.8}{2}t^2 + t v_0 \sin\theta + 10 = 0 \qquad \left(\begin{array}{l} v_0 = 150m \\ \theta = 45^\circ \end{array}\right)$$

$$-\frac{9.8}{2}t^2 + t\, 150 \frac{1}{\sqrt{2}} + 10 = 0$$

$t = 21.74$ $\qquad \frac{\sqrt{2}}{2} \qquad \frac{1}{\sqrt{2}}$

$x(21.74) = 2306$

Normal and Tangential components of acceleration

$$\vec{r}(t) = \langle \cos(t^2), \sin(t^2) \rangle$$

$$= \langle \cos(t\,t), \sin(t\,t) \rangle$$

$$\left[\langle \cos(\omega t), \sin(\omega t) \rangle \right]$$

$$\vec{r}'(t) = \langle -2t\sin(t^2),\ 2t\cos(t^2) \rangle$$

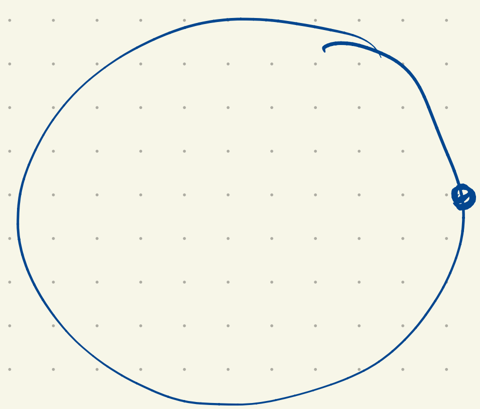
$$\|\vec{r}'(t)\|^2 = 4t^2\sin^2(t^2) + 4t^2\cos^2(t^2)$$

$$= 4t^2\left(\sin^2(t^2) + \cos^2(t^2)\right)$$

$$= 4t^2$$

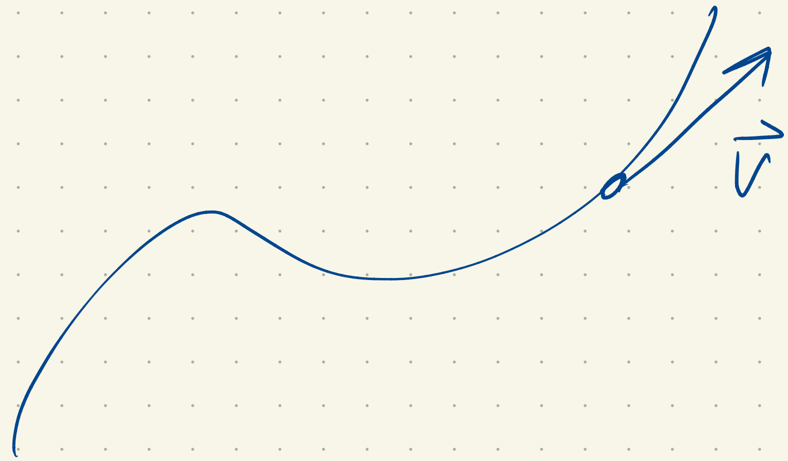
$\sin^2(\theta) + \cos^2(\theta) = 1$

$$\|\vec{r}'(t)\| = \sqrt{4t^2} = 2\sqrt{t^2} = 2|t|$$



$$\vec{v}(t) = v\,\vec{T}$$

speed

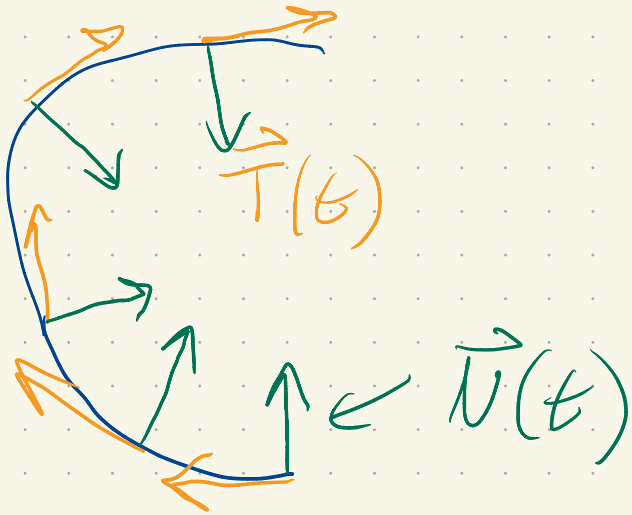


$\vec{r}(t)$

$\vec{r}\,'(t) \qquad \vec{T}(t) = \dfrac{\vec{r}\,'(t)}{\|\vec{r}\,'(t)\|} \rightarrow \vec{r}\,'(t) = \|\vec{r}\,'(t)\|\,\vec{T}(t)$

$$\vec{v}\,'(t) = v'\,\vec{T} + v\,\vec{T}\,'$$

$\|\vec{T}\|^2 = 1 \qquad \vec{T}\cdot\vec{T} = 1$



$$\frac{d}{dt}\vec{T}\cdot\vec{T} = 0$$

$$\vec{T}\cdot\vec{T}' + \vec{T}'\cdot\vec{T} = 0$$

$$\vec{T}\cdot\vec{T}' = 0$$

how much turning

$$\frac{\vec{r}\,'(t)}{\|\vec{r}\,'(t)\|}$$

$$\vec{N} = \frac{\vec{T}'}{\|\vec{T}'\|}$$

normal vector: points in the direction of turning

$\vec{r}(t)$ $\vec{T}(t)$

$\vec{r}\,'(t)/\|\vec{r}\,'(t)\|$

unit length