Section 3.4 (Acceleration, Velocity, Momentum, Force)

If $\vec{r}(t)$ describes position as a function of time

1) $\vec{v}(t) := \vec{r}\,'(t) = \frac{d}{dt}\vec{r}(t)$ is velocity

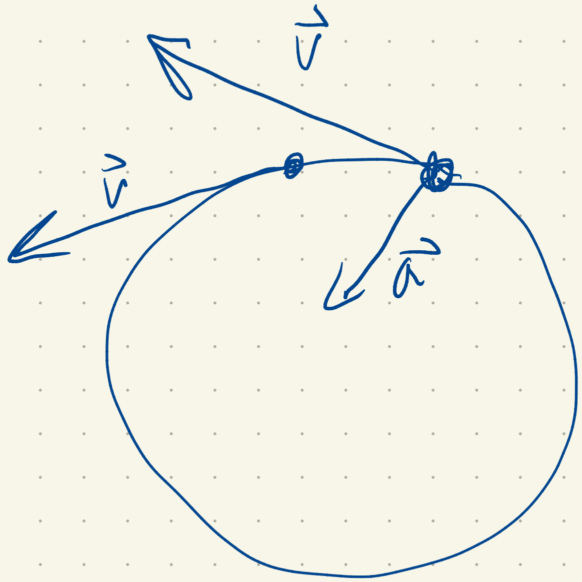
2) $\|\vec{v}(t)\|$ is speed

3) $\vec{a}(t) := \vec{v}\,'(t) = \vec{r}\,''(t)$ is acceleration

$$\vec{r}(t) = \langle \sin(2t), \tan(t), 1-t \rangle$$

$$\vec{r}\,'(t) = \langle 2\cos(2t), \sec^2(t), -1 \rangle \swarrow \vec{v}(t)$$

$$\vec{r}\,''(t) = \langle -4\sin(2t), 2\sec^2(t)\tan(t), 0 \rangle \swarrow \vec{a}(t)$$



$\vec{a}(t) = \langle -\cos(t), -\sin(t), -1 \rangle$

$\vec{r}(0) = \langle 5, 2, 2 \rangle$

$\vec{v}(0) = \vec{r}\,'(0) = \langle 0, 1, 3 \rangle$

Job: determine $\vec{r}(t)$.

$\vec{v}\,'(t) = \vec{a}(t)$

$$\vec{v}(t) = \int \vec{a}(t)\,dt + \vec{c}$$

$$= \int \langle -\cos(t), -\sin(t), -1 \rangle \,dt + \vec{c}$$

$$= \langle -\sin(t), \cos(t), -t \rangle + \vec{c}$$

$$\vec{v}(0) = \langle 0, 1, 3 \rangle$$

$$\langle 0, 1, 3 \rangle = \langle -\sin(0), \cos(0), -0 \rangle + \vec{c}$$

$$= \langle 0, 1, 0 \rangle + \vec{c}$$

$$\vec{c} = \langle 0, 0, 3 \rangle$$

$$\vec{v}(t) = \langle -\sin(t), \cos(t), -t \rangle + \langle 0, 0, 3 \rangle$$

$$= \langle -\sin(t), \cos(t), 3 - t \rangle$$

$\vdots$ (repeat!)

$$\vec{r}(t) = \left\langle 4 + \cos(t), 2 + \sin(t), 2 + 3t - \frac{t^2}{2} \right\rangle$$

$$\vec{r}\,'(t) = \langle -\sin(t), \cos(t), 3 - t \rangle$$

$$\vec{v}(0) = \langle -\sin(0), \cos(0), 3 - 0 \rangle$$

$$= \langle 0, 1, 3 \rangle$$

Newton's Second Law

1) Force is mass times acceleration

"eff equals em eh"

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momentum "total quantity of motion"

An object with mass m and velocity $\vec{v}$

has momentum $m\vec{v} = \vec{p}$ $\qquad [\vec{p}] = \frac{kg\ m}{s}$

$$\frac{d}{dt}\vec{p} = \vec{F}$$

force $\qquad [\vec{F}] = N = \frac{kg\ m}{s^2}$

$$\frac{d}{dt}\vec{p}(t) = \vec{F}(t)$$

constant mass

$$\frac{d}{dt} m\vec{v} = \vec{F}$$

$$m \frac{d}{dt}\vec{v} = \vec{F}$$

$$m\vec{a} = \vec{F}$$

If you know the applied force on an object

then you know $\vec{a} = \frac{\vec{F}}{m}$

If you also knew position and velocity at some moment in time then you can reconstruct position.

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projectile close to earth

Force due to gravity: mass times gravitational acceleration

$$m\vec{G}$$

$$\vec{G} = -9.8\hat{k} \ m/s^2$$

$$m\vec{a} = \overbrace{m\vec{G}}^{\vec{F}_g}$$

$$\vec{a} = \vec{G}$$

$$\vec{v}\,' = \vec{G}$$

$$\vec{v} = -9.8t\hat{k} + \vec{c}$$

$$\vec{r}\,'(t) = \vec{v}(t)$$

$$\vec{r}(t) = \int \vec{v}(t)\,dt + \vec{k}$$

$$= -\frac{9.8}{2}t^2\hat{k} + \vec{c}\,t + \vec{k}$$

$$\vec{v} = -9.8t\,\hat{k} + \vec{c}$$

$$\vec{v}(0) = 0 \cdot \hat{k} + \vec{c} \Rightarrow \vec{c} = \vec{v}_0$$

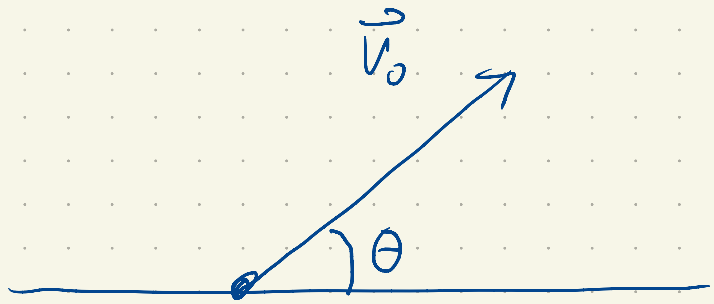
$$\vec{r} = -\frac{9.8}{2}t^2\,\hat{k} + \vec{v}_0 t + \vec{k}$$

$$\vec{r}(0) = 0\,\hat{k} + 0\,\vec{v}_0 + \vec{k}$$

$$\Rightarrow \vec{k} = \vec{r}_0$$

$$\vec{r}(t) = -\frac{9.8}{2}t^2\,\hat{k} + t\,\vec{v}_0 + \vec{r}_0$$

E.g. $\vec{r}_0 = 0$



$$\vec{v}_0 = v_0 \cos(\theta)\,\hat{\imath} + v_0 \sin(\theta)\,\hat{k}$$

$$\|\vec{v}_0\|^2 = v_0^2 \cos^2(\theta) + v_0^2 \sin^2(\theta)$$

$$= v_0^2 \left(\cos^2\theta + \sin^2\theta\right)$$

$$= v_0^2$$

$$\|\vec{v}_0\| = v_0$$