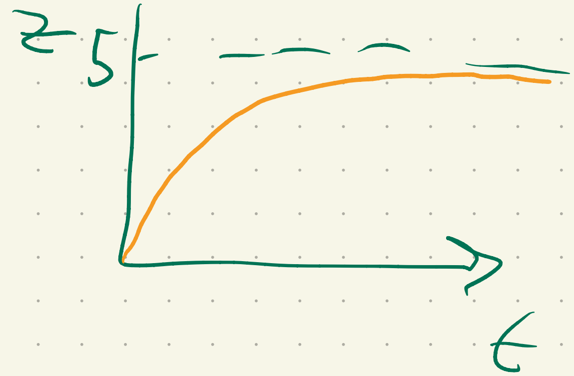
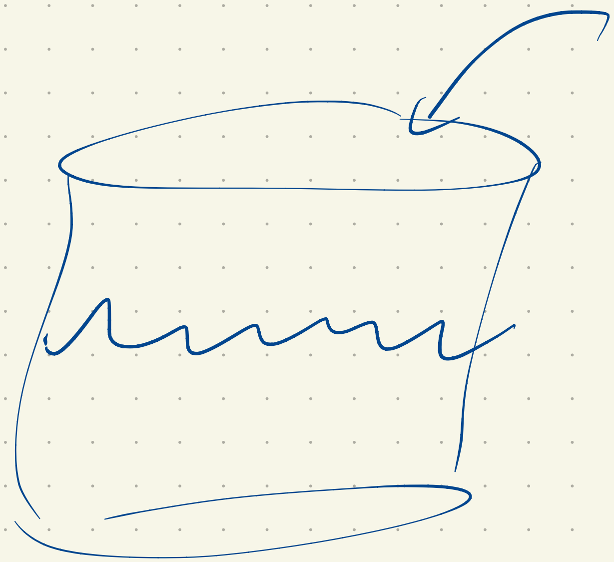
$\vec{r}(t) = \langle 50e^{-t}\cos t,\ 50e^{-t}\sin t,\ 5-5e^{-t} \rangle$



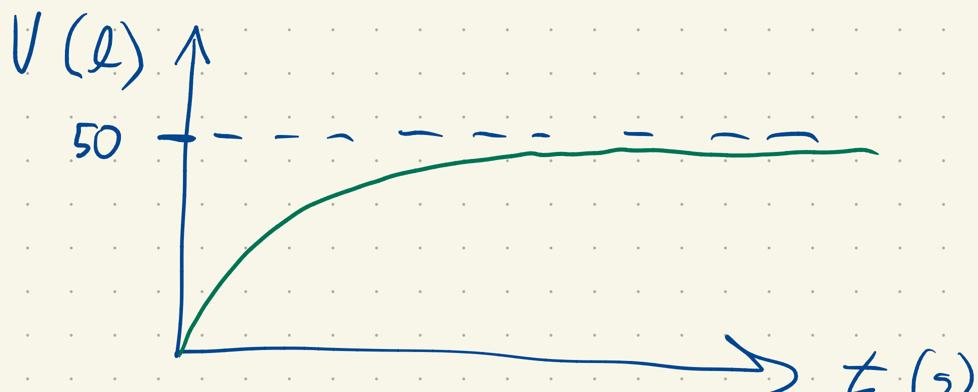
$\lim_{t\to\infty} \vec{r}(t)$

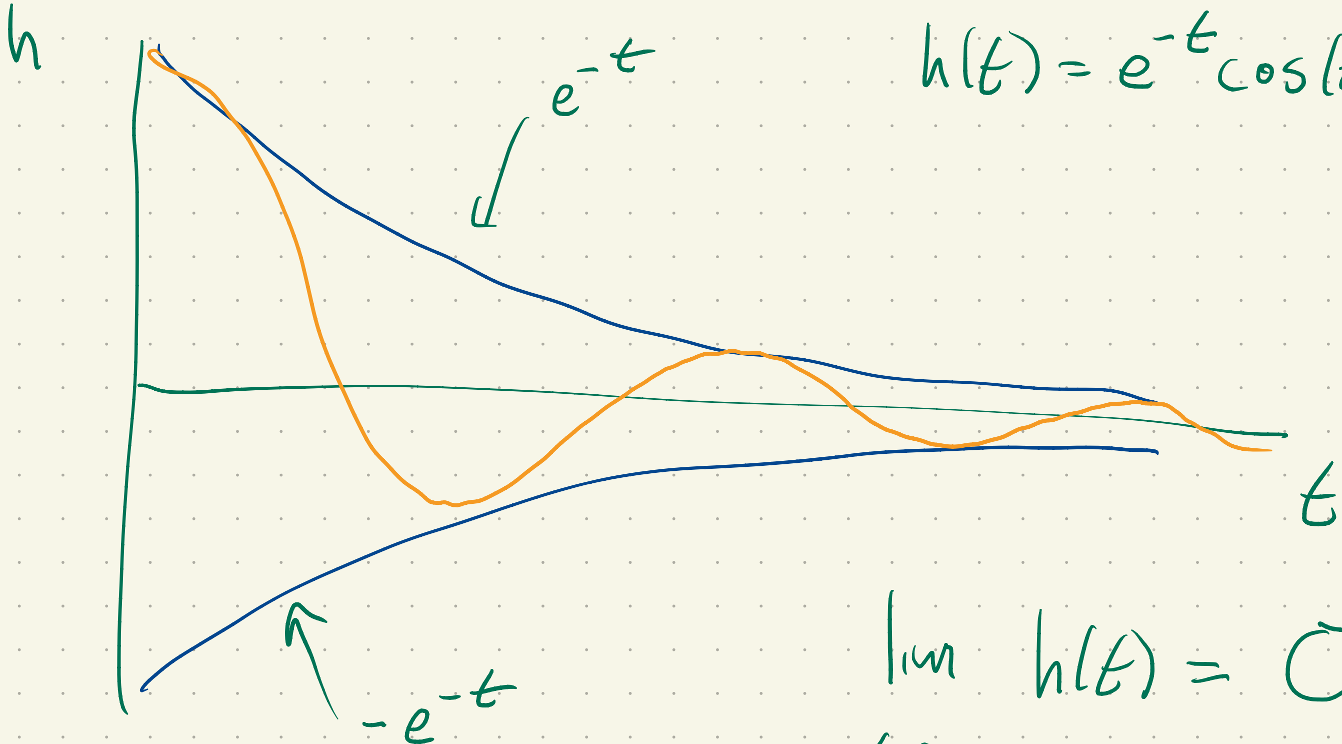


volume of water

$V(t) = 50(1-e^{-t})\ \ell$ (t in s)

$\boxed{\lim_{t\to\infty} V(t)}$

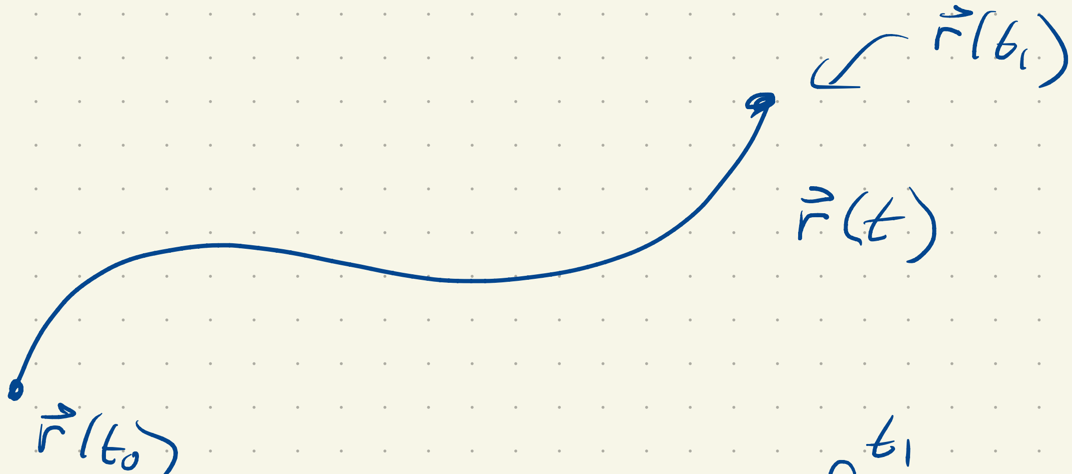




$$h(t) = e^{-t}\cos(t)$$

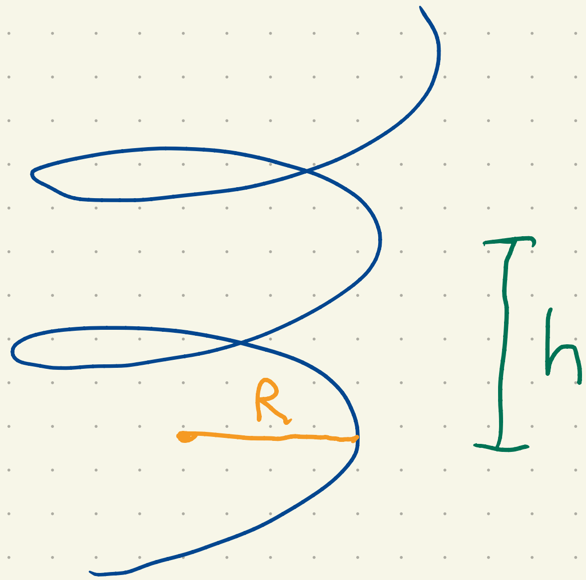
$$\lim_{t\to\infty} h(t) = 0$$

$$50\, e^{-t} \langle \cos t, \sin t, 0 \rangle$$



$$\text{arclength} = \int_{t_0}^{t_1} \|\vec{r}\,'(t)\|\,dt$$

helix 2 parameters: h, R



$$\vec{r}(t) = \left\langle R\cos\left(\frac{2\pi}{h}t\right), R\sin\left(\frac{2\pi}{h}t\right), t \right\rangle$$

$$\cos\left(\underbrace{\frac{2\pi}{h}\cdot\frac{2\pi}{h}}\right)$$

Let's compute arclength of one turn of helix

$$0 \leq t \leq h$$

$$\vec{r}\,'(t) = \left\langle -R\,\frac{2\pi}{h}\sin\left(\frac{2\pi}{h}t\right), \frac{2\pi}{h}R\cos\left(\frac{2\pi}{h}t\right), 1 \right\rangle$$

$$\|\vec{r}\,'(t)\|^2 = \left(\frac{2\pi}{h}R\right)^2\left(\sin^2\left(\frac{2\pi}{h}t\right) + \cos^2\left(\frac{2\pi}{h}t\right)\right) + 1^2$$

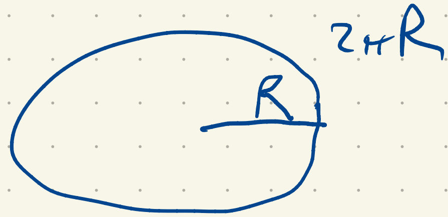
$$= \left(\frac{2\pi}{h}R\right)^2 + 1$$

$$\|\vec{r}\,'(t)\| = \sqrt{1 + \left(\frac{2\pi}{h}R\right)^2}$$

$$\text{arclength} = \int_0^h \sqrt{1+\left(\frac{2\pi}{h}R\right)^2}\,dt = h\sqrt{1+\left(\frac{2\pi}{h}R\right)^2}$$

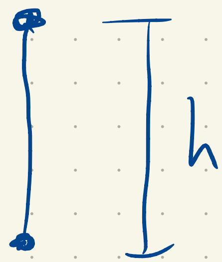
$$= \sqrt{h^2 + (2\pi R)^2}$$

$h = 0$



arclength $2\pi R$ ✓

$R = 0$



arclength h ✓

$\vec{r}(t) = \langle t, t^3, \sin(t) \rangle \qquad 0 \le t \le 1$

$\vec{r}\,'(t) = \langle 1, 3t^2, \cos(t) \rangle$

$$\|\vec{r}\,'(t)\|^2 = 1^2 + (3t^2)^2 + \cos^2(t)$$

$$= 1 + 9t^4 + \cos^2(t)$$

arclength $\displaystyle\int_0^1 \sqrt{1 + \cos^2(t) + 9t^4}\, dt$