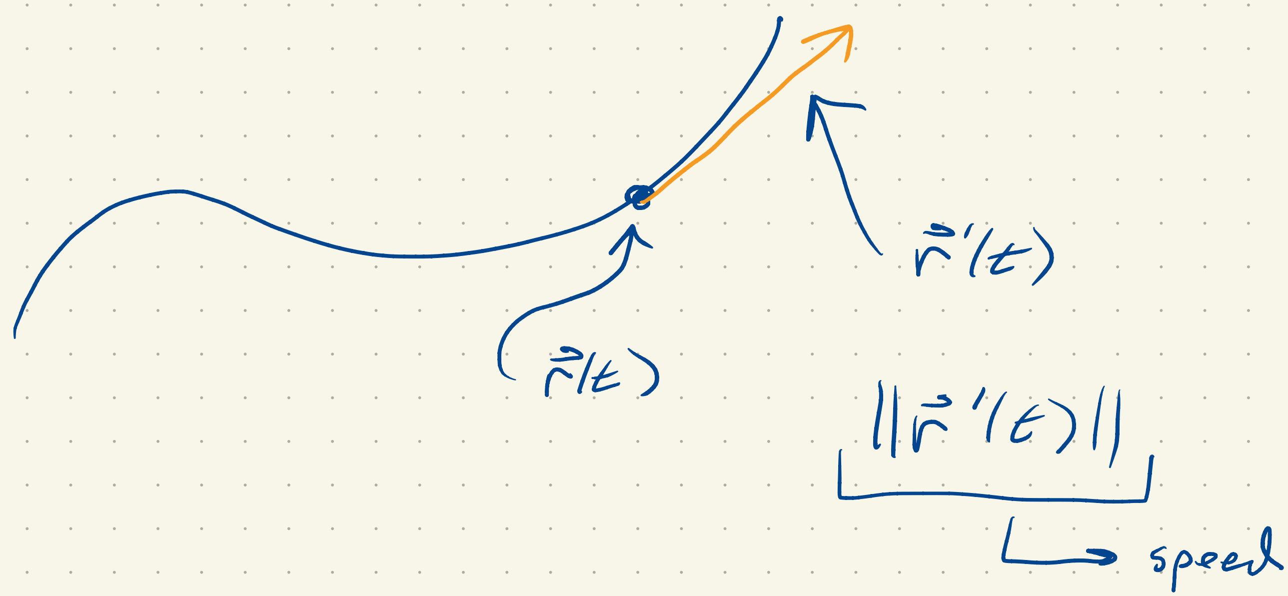
$$\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$$

$$\vec{r}(t) = \langle x'(t), y'(t), z'(t) \rangle$$



We can encode the direction of travel by a unit vector pointing in that direction.

$$\vec{T}(t) = \frac{\vec{r}\,'(t)}{\|\vec{r}\,'(t)\|} \qquad \text{(unit tangent vector)}$$

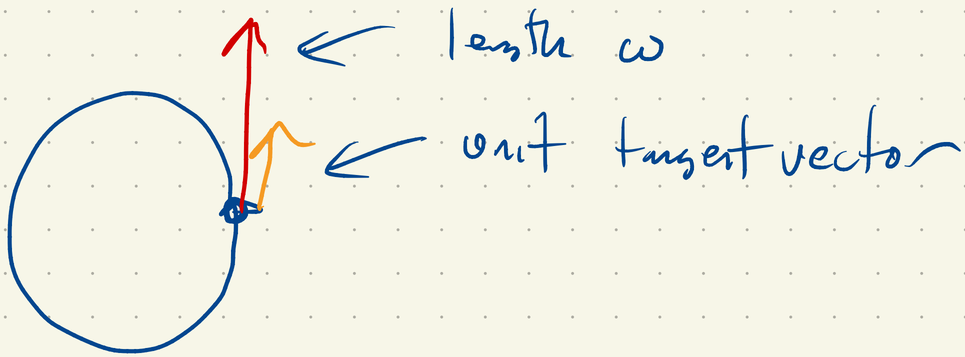
$$\vec{r}(t) = \langle \cos(\omega t), \sin(\omega t) \rangle \qquad (\omega > 0)$$

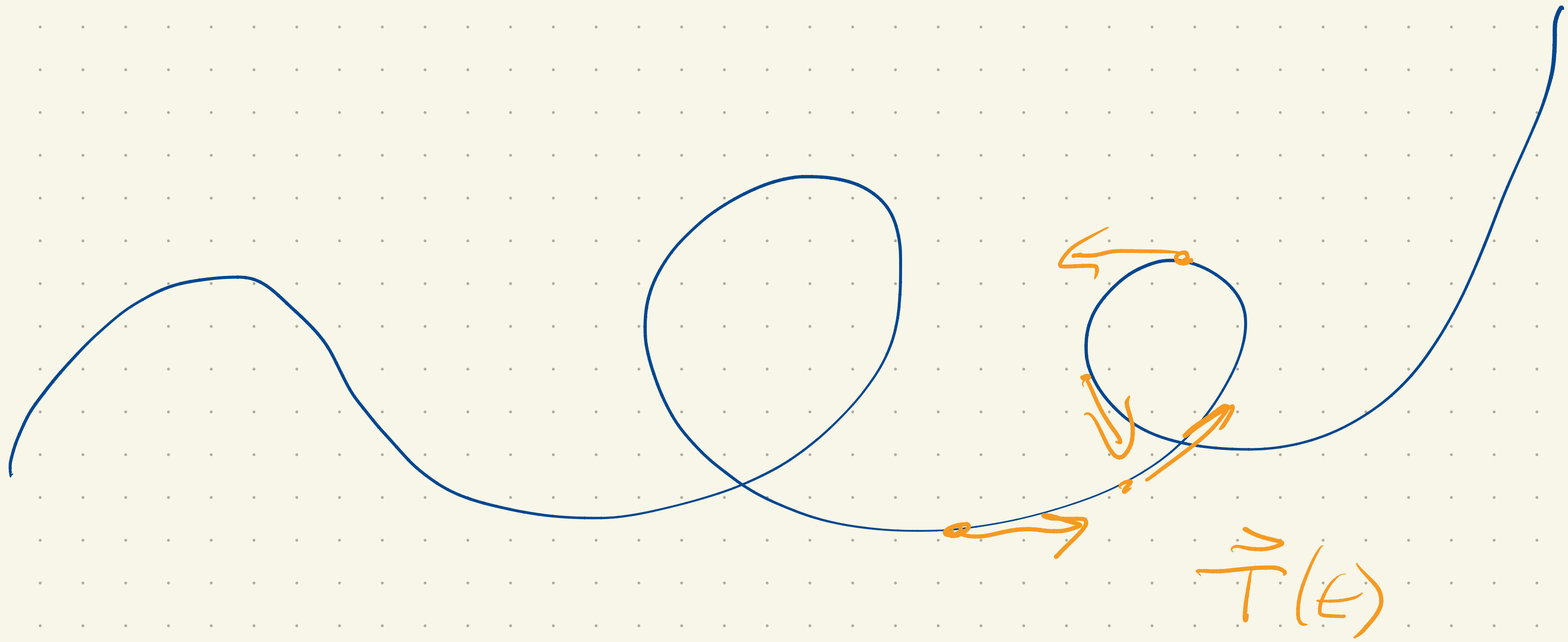
$$\vec{r}\,'(t) = \langle -\omega \sin(\omega t), \omega \cos(\omega t) \rangle$$

$$= \omega \langle -\sin(\omega t), \cos(\omega t) \rangle$$

$$\vec{T}(t) = \frac{\vec{r}\,'(t)}{\|\vec{r}\,'(t)\|} = \frac{\vec{r}\,'(t)}{\omega} = \langle -\sin(\omega t), \cos(\omega t) \rangle$$

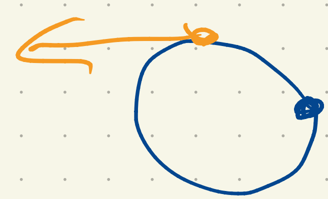
$$\|\vec{r}\,'(t)\| = \sqrt{\omega^2}$$
$$= |\omega|$$





Tangent lines to curves

$$\vec{r}(t) = \langle \cos(2\pi t), \sin(2\pi t) \rangle$$



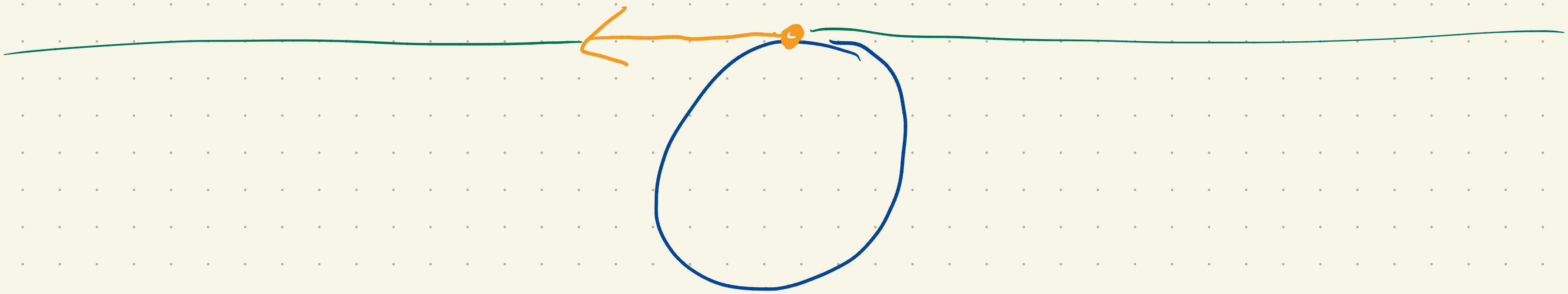
$$t = \frac{1}{4}$$

$$\vec{r}\left(\tfrac{1}{4}\right) = \langle 0, 1 \rangle$$

$$\vec{r}\,'\left(\tfrac{1}{4}\right) =$$

$$\vec{r}\,'(t) = \langle -2\pi \sin(2\pi t),\ 2\pi \cos(2\pi t) \rangle$$

$$\vec{r}\,'(1/4) = \langle -2\pi,\ 0 \rangle$$



$$\boxed{\vec{\ell}(t) = \vec{r}\left(\tfrac{1}{4}\right) + \vec{r}\,'\left(\tfrac{1}{4}\right)\left(t - \tfrac{1}{4}\right)}$$

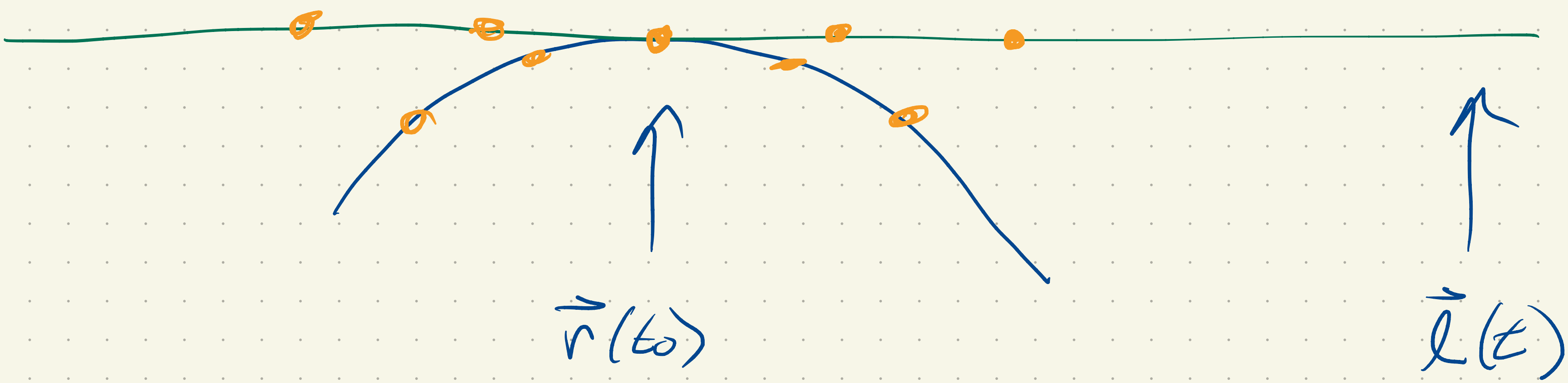
$$= \left[\vec{r}\left(\tfrac{1}{4}\right) - \tfrac{1}{4}\vec{r}\,'\left(\tfrac{1}{4}\right)\right] + \vec{r}\,'\left(\tfrac{1}{4}\right) t$$

$$\vec{\ell}\left(\tfrac{1}{4}\right) = \vec{r}\left(\tfrac{1}{4}\right)$$

$$\vec{\ell}\,'\left(\tfrac{1}{4}\right) = \vec{r}\,'\left(\tfrac{1}{4}\right)$$

$$= \vec{r}_0 + \vec{v}\, t$$

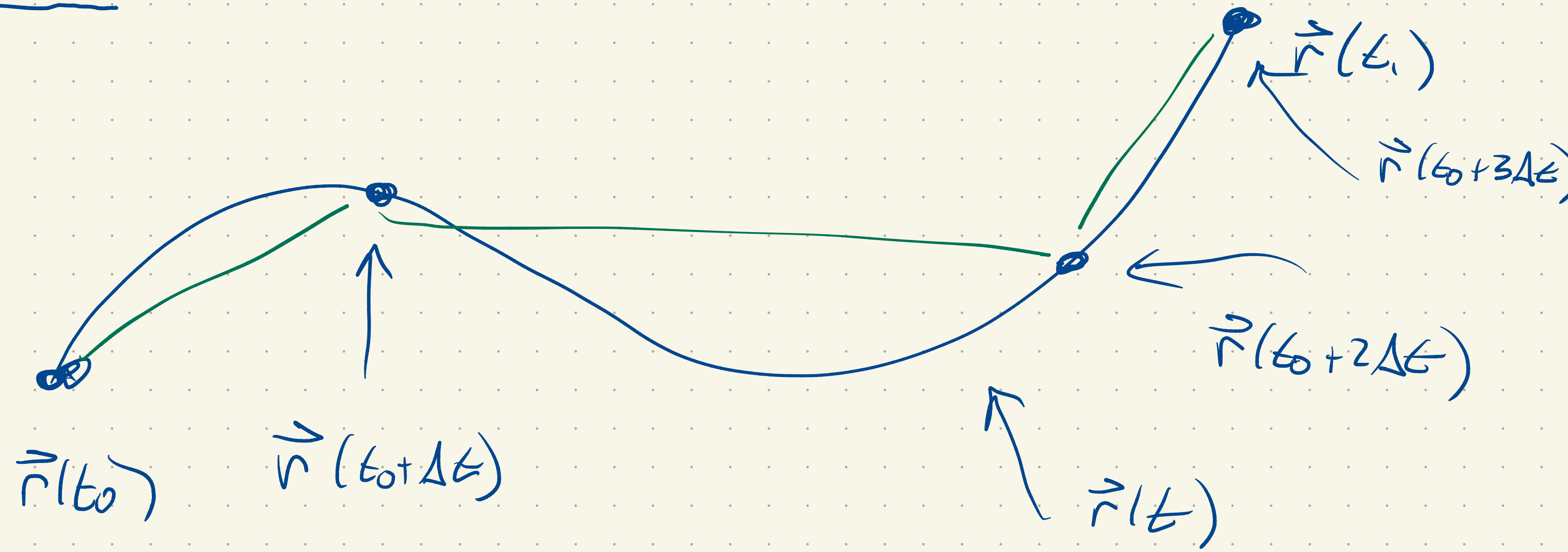
$$\boxed{\vec{r}(t) = \vec{r}_0 + \vec{v}t}$$



$\vec{l}(t) \longrightarrow$ linear approximation of $\vec{r}(t)$ at $t = t_0$

$$\boxed{\vec{l}(t) = \vec{r}(t_0) + \vec{r}\,'(t_0)(t - t_0)}$$

Arclength



$$\|\vec{r}(t_1) - \vec{r}(t_0)\|$$

$$\|\vec{r}(t_0+\Delta t) - \vec{r}(t_0)\| + \|\vec{r}(t_0+2\Delta t) - \vec{r}(t_0+\Delta t)\| + \|\vec{r}(t_1) - \vec{r}(t_0+2\Delta t)\|$$

$\rightarrow$ approximate length

$$\frac{\vec{r}(t_0+\Delta t)-\vec{r}(t_0)}{\Delta t} \approx \vec{r}\,'(t_0)$$

(better approx if Δt is small)

$$\vec{r}(t_0+\Delta t)-\vec{r}(t_0) \approx \vec{r}\,'(t_0)\,\Delta t$$

$$\|\vec{r}(t_0+\Delta t)-\vec{r}(t_0)\| \approx \|\vec{r}\,'(t_0)\|\,\Delta t$$

approx length:

$$\|\vec{r}\,'(t_0)\|\Delta t + \|\vec{r}\,'(t_0+\Delta t)\|\Delta t + \|\vec{r}\,'(t_0+2\Delta t)\|\Delta t$$

arclength $\displaystyle\int_{t_0}^{t_1} \|\vec{r}\,'(t)\|\,dt$

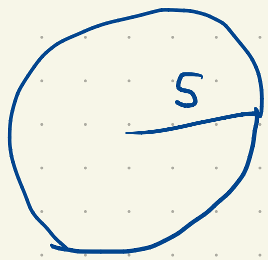
$[\vec{r}] = m$ $\quad [dt] = s$

$[t] = s$

$[\vec{r}\,'(t)] = m/s$

$\vec{r}(t) = \langle 5\cos(t), 5\sin(t) \rangle$

$t_0 = 0, \quad t_1 = 2\pi$



arclength should be $2\pi \cdot 5 = 10\pi$

$\vec{r}\,'(t) = \langle -5\sin(t), 5\cos(t) \rangle$

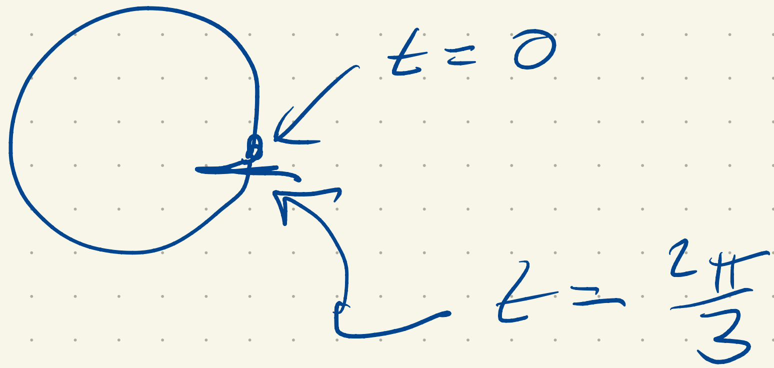
$\int_0^{2\pi} \|\vec{r}\,'(t)\| \, dt$

$\|\vec{r}\,'(t)\| = 5$

$\|\vec{r}\,'(t)\|^2 = 25\sin^2(t) + 25\cos^2(t)$

$= \int_0^{2\pi} 5 \, dt = 2\pi \cdot 5 = 10\pi$

$$\vec{r}(t) = \langle 5\cos(3t),\ 5\sin(3t) \rangle$$



$t_0 = 0, \quad t_1 = \frac{2\pi}{3}$

$$\text{arclength} = \int_0^{2\pi/3} \|\vec{r}\,'(t)\|\, dt$$

$$\vec{r}\,'(t) = \langle -15\sin(3t),\ 15\cos(3t) \rangle$$

$$\|\vec{r}\,'(t)\| = 15$$

$$\int_0^{2\pi/3} 15\, dt = 15 \cdot \frac{2\pi}{3} = 5 \cdot 2\pi = 10\pi$$

Fact: arclength is independent of how the curve is parameterized (so long as there is no backtracking)

helix