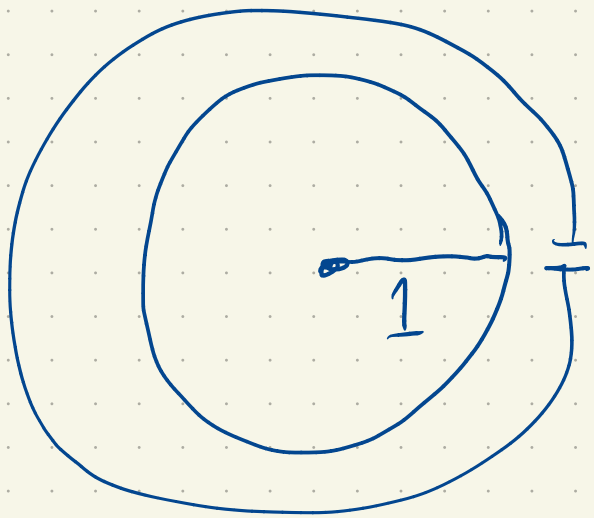
$\vec{r}(0) = \langle 1, 0 \rangle$

$\vec{r}\left(\frac{2\pi}{\omega}\right) = \langle 1, 0 \rangle$

$x \qquad\qquad y$

$\vec{r}(t) = \langle \cos(\omega t), \underline{\sin(\omega t)} \rangle$

$\vec{r}\,'(t) = \langle -\omega \sin(\omega t), \omega \cos(\omega t) \rangle$



dist 2π

$x^2 + y^2 =$

$\cos(\omega t)^2 + \sin(\omega t)^2 = 1$

$[t] = s$

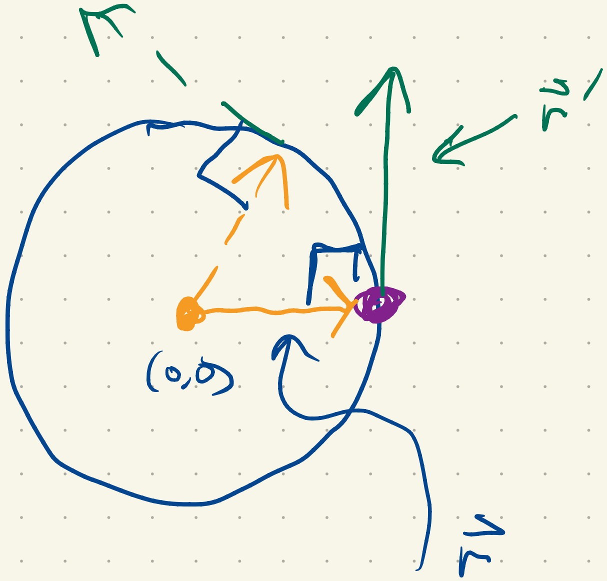
$[x] = m$

$2\pi\, m$

$\frac{2\pi}{\omega} s$

speed $\frac{2\pi m}{(2\pi)/\omega\, s}$

$\omega \frac{m}{s}$



$$[\vec{r}\,] = m/s$$

$$\|\vec{r}\,'\|$$

$$\vec{r}\,' = \langle -\omega \sin(\omega t), \ \omega \cos(\omega t) \rangle$$

$$\|\vec{r}\,'\|^2 = \omega^2 \sin^2(\omega t) + \omega^2 \cos^2(\omega t)$$

$$= \omega^2 \left[\sin^2(\omega t) + \cos^2(\omega t) \right]$$

$$= \omega^2$$

$$(\omega > 0)$$

$$\|\vec{r}\,'\| = |\omega| = \omega$$

$$\vec{r}(t) \cdot \vec{r}\,'(t) \stackrel{?}{=} 0$$

$$\vec{r}(t) = \langle \cos(\omega t), \sin(\omega t) \rangle$$

$$\vec{r}\,'(t) = \langle -\omega \sin(\omega t), \omega \cos(\omega t) \rangle$$

$$\vec{r} \cdot \vec{r}\,' = -\omega \sin(\omega t)\cos(\omega t) + \omega \sin(\omega t)\cos(\omega t)$$

$$= 0$$

$$\vec{r}(t) = \vec{r}_0 + \vec{v}\, t$$

$$\vec{r}_0 = \langle x_0, y_0, z_0 \rangle$$

$$\vec{v} = \langle a, b, c \rangle$$

$$\vec{r}\,'(t) \stackrel{?}{=}$$

$$\vec{r}(t) = \langle x_0 + at, y_0 + bt, z_0 + ct \rangle$$

$$\vec{r}'(t) = \langle a, b, c \rangle$$

$$= \vec{v}$$

$$\vec{r}'(t) = \frac{d}{dt}\left(\vec{r}_0 + \vec{v}t\right)$$

$$= 0 + \vec{v}\frac{d}{dt}t$$

$$= \vec{v} \cdot 1$$

Rules

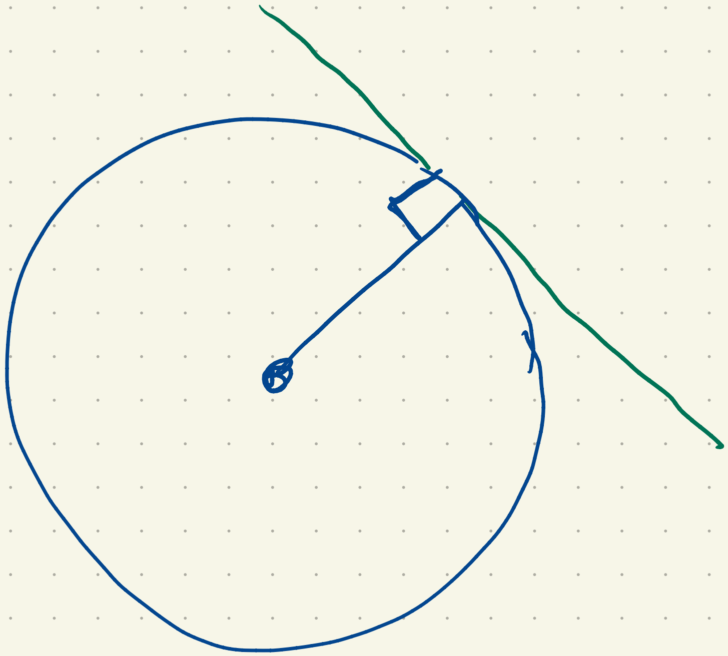
$$\frac{d}{dt}\left(\vec{r}(t)+\vec{s}(t)\right)=\frac{d}{dt}\vec{r}(t)+\frac{d}{dt}\vec{s}(t)$$

$$\frac{d}{dt}\left(f(t)\,\vec{v}(t)\right)=f'(t)\,\vec{v}(t)+f(t)\,\vec{v}\,'(t)$$

$$\frac{d}{dt}\left(\vec{r}(t)\cdot\vec{s}(t)\right)=\vec{r}\,'(t)\cdot\vec{s}(t)+\vec{r}(t)\cdot\vec{s}\,'(t)$$

$$\frac{d}{dt}\left(\vec{r}(t)\times\vec{s}(t)\right)=\vec{r}\,'(t)\times\vec{s}(t)+\vec{r}(t)\times\vec{s}\,'(t)$$

$$\frac{d}{dt}\,\vec{r}\left(f(t)\right)=\vec{r}\,'\left(f(t)\right)f'(t)$$



$\vec{r}(t)$

$\|\vec{r}(t)\| = 5$ for all t.

$\vec{r}\,'(t) \cdot \vec{r}(t) = 0$

$\|\vec{r}(t)\|^2 = 25$

$\vec{r}(t) \cdot \vec{r}(t) = 25$

$$\frac{d}{dt}\left(\vec{r}(t) \cdot \vec{r}(t)\right) = \frac{d}{dt} 25$$

$$\vec{r}\,'(t) \cdot \vec{r}(t) + \vec{r}(t) \cdot \vec{r}\,'(t) = 0$$

$$2\,\vec{r}(t) \cdot \vec{r}\,'(t) = 0$$

$\vec{r}(t) \cdot \vec{r}'(t) = 0$

Integration

$$\int_0^2 x^3\,dx = \frac{x^4}{4}\Big|_0^2 = \frac{2^4}{4} - \frac{0^4}{4} = \frac{16}{4} = 4$$

$f(x) = x^3$

$F(x) = \frac{x^4}{4}$ ($F(x)$ is an antiderivative of $f(x)$)

$F'(x) = x^3$

$$\underline{\int_0^2 F'(x)\,dx} = \underline{F(2) - F(0)}$$

rate of change of F

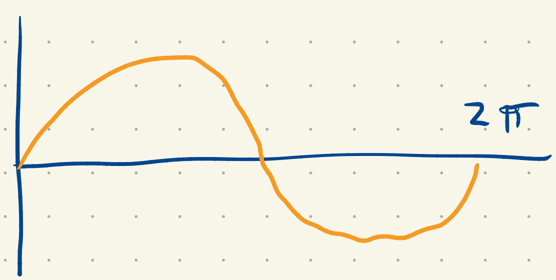
net change in F

ΔF

If you integrate a rate of change you get a net change

$$\int_0^{2\pi} \sin(x)\,dx = -\cos(x)\Big|_0^{2\pi} = -\cos(2\pi) + \cos(0)$$

$$= -1 + 1 = 0$$



$$\vec{s}(t) = \langle x(t), y(t), z(t) \rangle$$

$$\int_a^b \vec{s}(t)\,dt := \left\langle \int_a^b x(t)\,dt, \int_a^b y(t)\,dt, \int_a^b z(t)\,dt \right\rangle$$

$$\vec{r}\,'(t) \qquad (\vec{v}(t))$$

$$\int_a^b \vec{r}\,'(t)\,dt = \vec{r}(b) - \vec{r}(a)$$

$$\int_a^b \vec{r}\,'(t)\,dt = \left\langle \int_a^b x'(t)\,dt, \int_a^b y'(t)\,dt, \int_a^b z'(t)\,dt \right\rangle$$

$$= \langle x(b) - x(a),\ y(b) - y(a),\ z(b) - z(a) \rangle$$

$$= \langle x(b), y(b), z(b) \rangle - \langle x(a), y(a), z(a) \rangle$$

$$= \vec{r}(b) - \vec{r}(a)$$

Suppose we know $\vec{r}\,'(t) = \vec{v}(t) = \langle 5, 2 - 9.8t, 0 \rangle$

$\vec{r}(0) = \langle 1, 3, 2 \rangle$

Job: determine $\vec{r}(t)$

$$\int_0^t v(t) = \int_0^t \vec{r}\,'(t)\,dt = \vec{r}(t) - \vec{r}(0)$$

$$\vec{r}(t) = \vec{r}(0) + \int_0^t v(s)\,ds$$

$$= \langle 1, 3, 2 \rangle + \int_0^t \langle 5,\ 2 - 9.8t,\ 0 \rangle\,dt$$

$$= \langle 1, 3, 2 \rangle + \left\langle \int_0^t 5\,d\overset{s}{\cancel{t}},\ \int_0^t (2 - 9.8\overset{s}{\cancel{t}})\,d\overset{s}{\cancel{t}},\ \int_0^t 0\,dt \right\rangle$$

$$= \langle 1, 3, 2 \rangle + \left\langle 5t,\ 2\overset{s}{\cancel{t}} - \frac{9.8}{2}\overset{s}{\cancel{t}}^2 \Big|_0^t,\ 0 \right\rangle$$

$$= \langle 1, 3, 2 \rangle + \left\langle 5t,\ 2t - \frac{9.8}{2}t^2,\ 0 \right\rangle$$

$$= \left\langle 5t + 1,\ 3 + 2t - \frac{9.8}{2}t^2,\ 2 \right\rangle$$

$$\int \vec{s}(t)\,dt$$ ← indefinite integral

just some antiderivative

of $\vec{s}$

$$\int \vec{r}\,'(t)\,dt = \vec{r}(t) - \vec{c}$$

$$\vec{r}(t) = \vec{c} + \int \vec{r}\,'(t)\,dt$$

$$= \vec{c} + \int \langle 5, 2-9.8t, 0 \rangle\,dt$$

$$= \vec{c} + \langle 5t, 2t - \frac{9.8}{2}t^2, 7 \rangle$$

$$\vec{r}(0) = \langle 1, 3, 2 \rangle$$

$$\langle 1, 3, 2 \rangle = \vec{c} + \langle 0, 0, 7 \rangle$$

$$\vec{c} = \langle 1, 3, 2-7 \rangle$$

$$= \langle 1, 3, -5 \rangle$$

$$\vec{r}(t) = \langle 1, 3, \underline{-5} \rangle + \langle 5t, 2t - \frac{9.8}{2}t^2, \underline{7} \rangle$$

$$= \langle 1 + 5t, 3 + 2t - \frac{9.8}{2}t^2, 2 \rangle$$