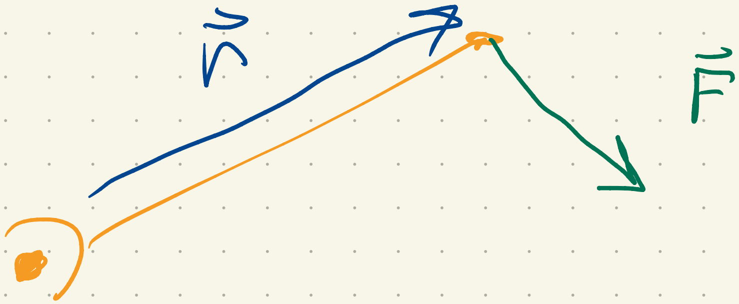


$\vec{F}$ is pointing in direction $\langle 1, -1, 0 \rangle$

total force 100 N

$\langle \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0 \rangle$

m $\quad$ N

$\downarrow \quad \downarrow$

$\vec{r} \times \vec{F}$

$$\begin{array}{ccc|l} \hat{i} & \hat{j} & \hat{k} & \\ 0.5 & 0.2 & 0 & \leftarrow \vec{r} \\ \hline 1 & -1 & 0 & \leftarrow \end{array}$$

$$\frac{100}{\sqrt{2}} \quad \frac{-100}{\sqrt{2}} \quad 0$$

$\vec{F} = \langle 100, -100, 0 \rangle$

$\|\vec{F}\| = \sqrt{100^2 + 100^2 + 0^2}$

$= \sqrt{2}\,100$

"velocity"

$$\vec{r}(t) = \vec{r}_0 + t\,\vec{v}$$

$\vec{w} = 2\vec{v}$

$$\vec{r}_0 = \langle x_0, y_0, z_0 \rangle$$

$$\vec{v} = \langle a, b, c \rangle$$

$\vec{r}_0 + 2\vec{v}$

$\vec{r}_0 + \vec{v}$

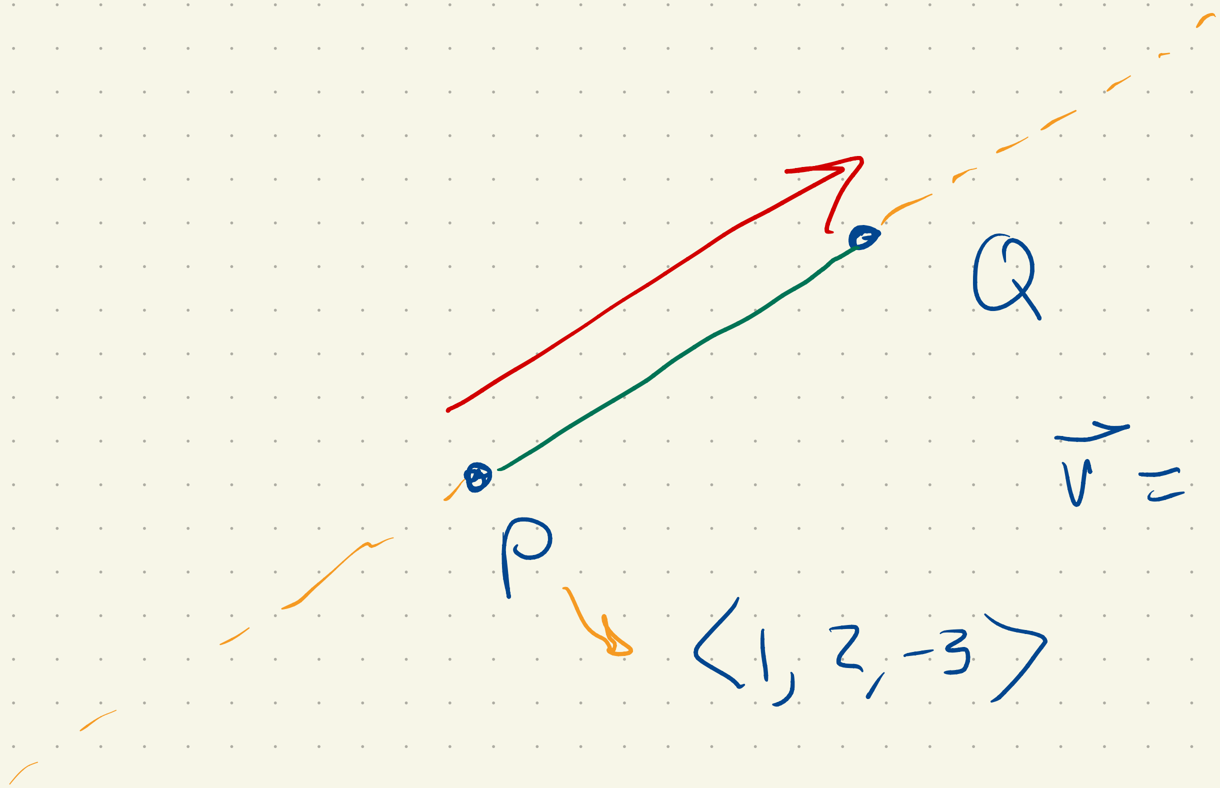
$\vec{r}_0$

$$\vec{r}(t) = \langle \cos t, \sin t, 2t \rangle$$

helix

Task: Find a vector-valued function (space curve) parameterizing the line segment from

$P(1,2,-3)$ to $Q(4,1,6)$



$(5,7,1) \to \langle 5,7,1\rangle$

$\langle 1,2,-3\rangle$

$$\vec{v} = \overrightarrow{PQ} = \langle 4-1,\ 1-2,\ 6-(-3)\rangle$$

$$= \langle 3,-1,9\rangle$$

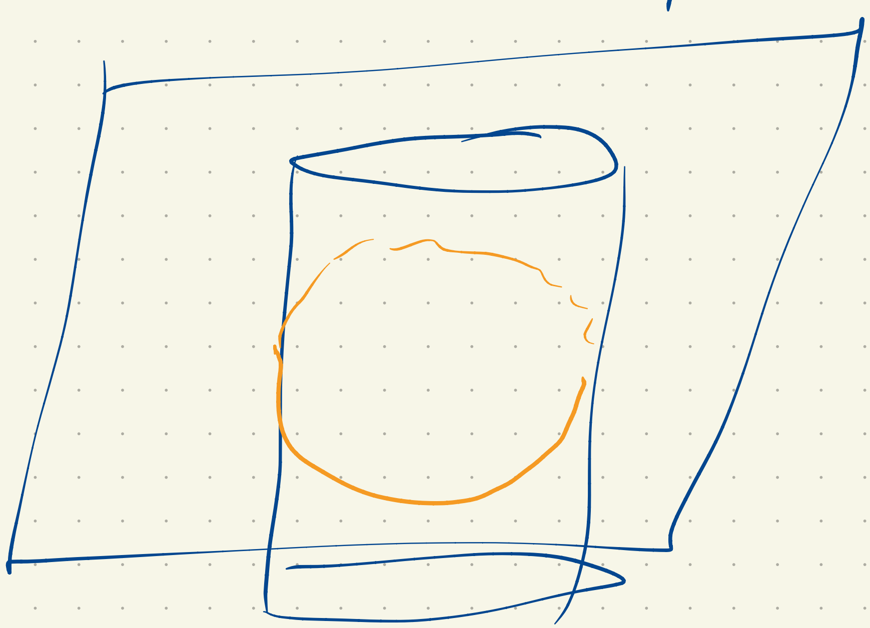
$$\vec{r}(t) = \langle 1,2,-3\rangle + t\langle 3,-1,9\rangle$$

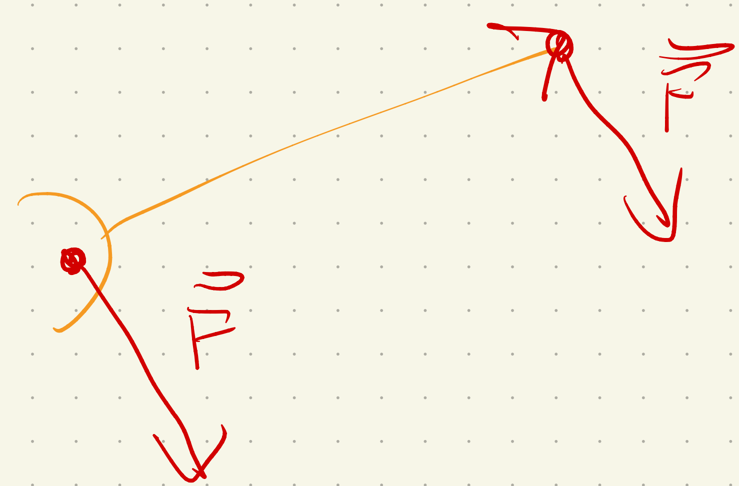
$$0 \le t \le 1 \qquad \left(\begin{aligned} \vec{r}(0) &= P \\ \vec{r}(1) &= Q \end{aligned} \right)$$

Parameterize the curve that is the intersection of

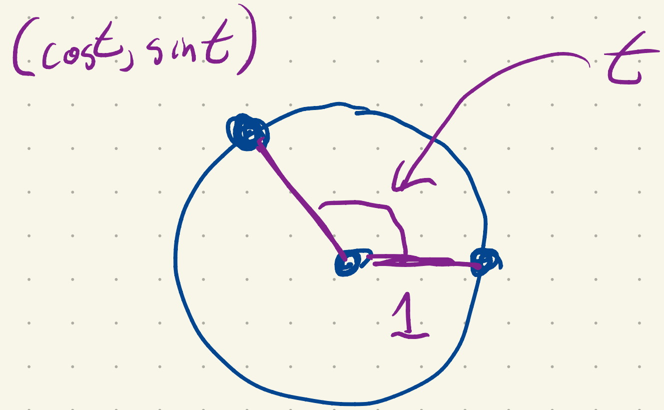
cylinder $x^2+y^2=1$

plane $z = x+y$ $\quad (x+y-z=0)$





$$\langle \underset{x}{\cos(t)}, \ \underset{y}{\sin(t)}, \qquad \rangle$$



$$x^2 + y^2 = 1$$

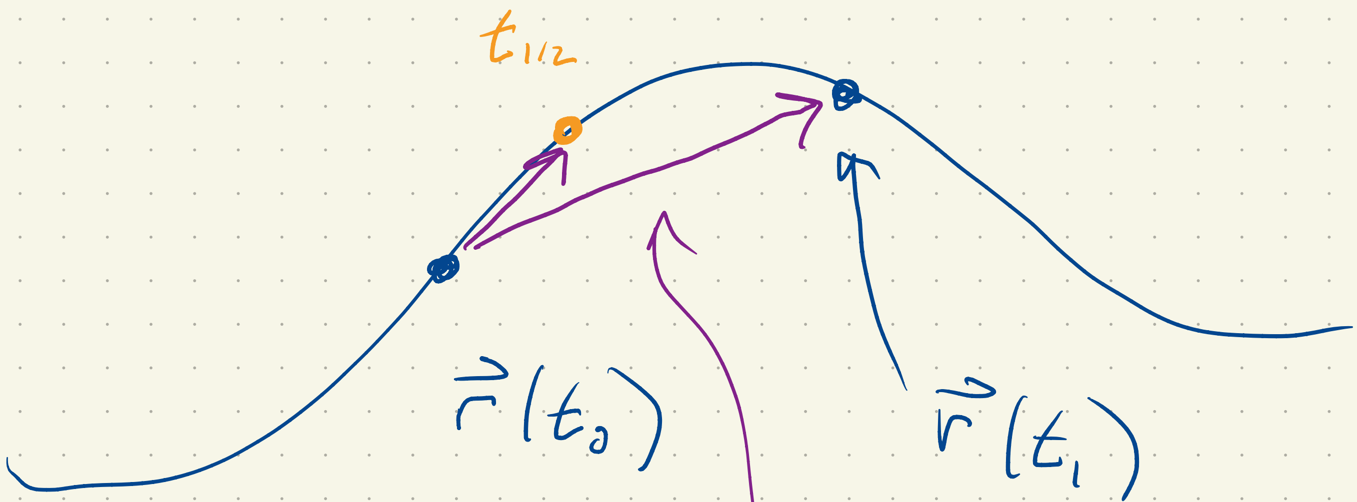
$$z = x + y$$

$$\vec{r}(t) = \langle \cos(t), \sin(t), \cos(t) + \sin(t) \rangle$$

Section 3.2

Derivatives and Integrals of vector-valued functions

$$\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$$



$\vec{r}(t_1) - \vec{r}(t_0)$

change in position

[displacement of position between times t_0 and t_1

change in time: $t_1 - t_0$

average velocity over time interval t_0 to t_1

$$\frac{\vec{r}(t_1) - \vec{r}(t_0)}{t_1 - t_0}$$

$\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$

$$\vec{r}\,'(t_0) = \lim_{t_1 \to t_0} \frac{\vec{r}(t_1) - \vec{r}(t_0)}{t_1 - t_0}$$

$$= \lim_{t_1 \to t_0} \left\langle \frac{x(t_1) - x(t_0)}{t_1 - t_0}, \frac{y(t_1) - y(t_0)}{t_1 - t_0}, \frac{z(t_1) - z(t_0)}{t_1 - t_0} \right\rangle$$

$$= \left\langle \lim_{t_1 \to t_0} \frac{x(t_1) - x(t_0)}{t_1 - t_0}, \right.$$

$$\lim_{t_1 \to t_0} \frac{y(t_1) - y(t_0)}{t_1 - t_0},$$

$$\left. \lim_{t_1 \to t_0} \frac{z(t_1) - z(t_0)}{t_1 - t_0} \right\rangle$$

$$= \langle x'(t_0), y'(t_0), z'(t_0) \rangle$$

$$\vec{r}\,'(t_0) = \langle x'(t_0), y'(t_0), z'(t_0) \rangle$$

$$\vec{r}\,'(t) = \langle x'(t), y'(t), z'(t) \rangle$$

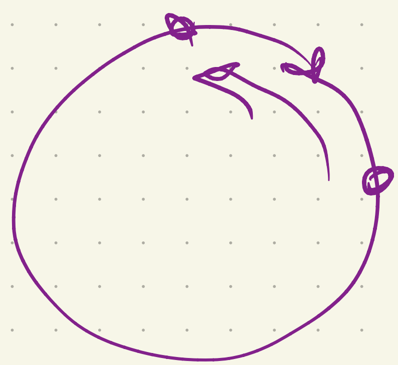
$$\vec{r}(t) = \langle \cos(\omega t), \sin(\omega t) \rangle$$

$$\vec{r}(0) = \langle 1, 0 \rangle$$

$$\vec{r}\Big(\underbrace{\frac{2\pi}{\omega}}_{\text{time to go once around}}\Big) = \left\langle \cos\left(\omega \frac{2\pi}{\omega}\right), \sin\left(\omega \frac{2\pi}{\omega}\right) \right\rangle$$

$$= \langle \cos(2\pi), \sin(2\pi) \rangle$$

$$= \langle 1, 0 \rangle$$



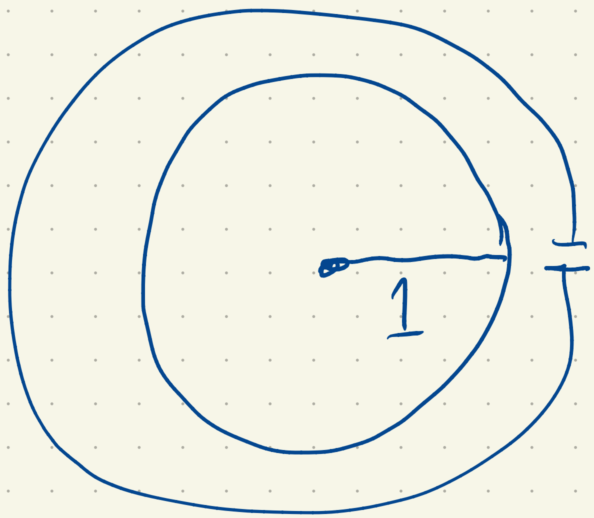
$\vec{r}(0) = \langle 1, 0 \rangle$

$\vec{r}\left(\frac{2\pi}{\omega}\right) = \langle 1, 0 \rangle$

$x \qquad y$

$$\vec{r}(t) = \langle \cos(\omega t), \underline{\sin(\omega t)} \rangle$$

$$\vec{r}\,'(t) = \langle -\omega \sin(\omega t), \omega \cos(\omega t) \rangle$$



dist 2π

$x^2 + y^2 =$

$\cos(\omega t)^2 + \sin(\omega t)^2 = 1$

$[t] = s$

$[x] = m$

$2\pi\, m$

$\frac{2\pi}{\omega} s$

speed $\frac{2\pi m}{(2\pi)/\omega\, s}$

$\omega \frac{m}{s}$