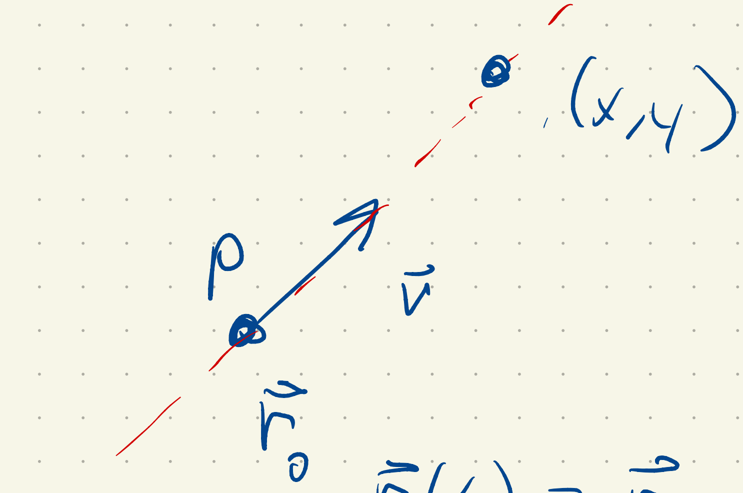
$P(x_0, y_0)$

$\langle x_0, y_0 \rangle$

$\vec{r}_0$

$\vec{v} = \langle v_1, v_2 \rangle$



$$\vec{r}(t) = \vec{r}_0 + t\vec{v}$$ "vector form"

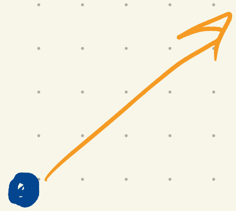
$$x = x_0 + tv_1$$
$$y = y_0 + tv_2$$ "parametric form"

$\vec{r}_0 \longrightarrow$ starting point

$\vec{v} \longrightarrow$ velocity

$t \longrightarrow$ time

"parameter" labels



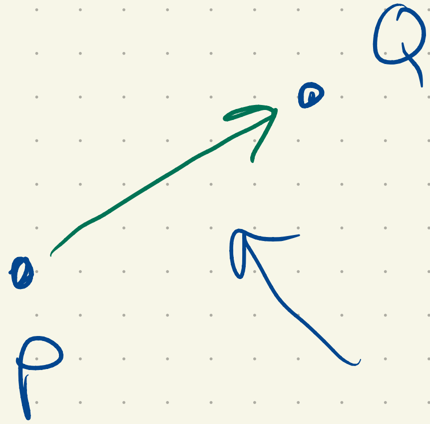
$\vec{v} = \langle v_1, v_2, v_3 \rangle$

$\vec{r}_0 = \langle x_0, y_0, z_0 \rangle$

$$\vec{r}(t) = \vec{r}_0 + t\,\vec{v}$$

$P(2,-4,1) \qquad Q(8,-3,-1)$

vector form of line between these two points



$$\overrightarrow{PQ} = \langle 8-2,\ -3-(-4),\ -1-1 \rangle$$

$$= \langle 6,\ 1,\ -2 \rangle$$

$$\vec{r}(t) = \langle 2, -4, 1 \rangle + t\langle 6, 1, -2 \rangle$$

$$= \langle 2+6t,\ -4+t,\ 1-2t \rangle$$

$$x = 2+6t$$
$$y = -4+t$$
$$z = 1-2t$$

Symmetric form of the line

$x = 2 + 6t \longrightarrow \dfrac{x-2}{6} = t$

$y = -4 + t \longrightarrow \dfrac{y+4}{1} = t$

$z = 1 - 2t \longrightarrow \dfrac{z-1}{-2} = t$

$$\frac{x-2}{6} = \frac{y+4}{1} = \frac{z-1}{-2}$$

symmetric form

$$y = mx + b$$

$$\vec{r}(t) = \langle 2, -4, 1 \rangle + t\langle 6, 1, -2 \rangle$$

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

$\vec{r}_0 = \langle x_0, y_0, z_0 \rangle$ point on line

$\vec{v} = \langle a, b, c \rangle$ points in direction of line

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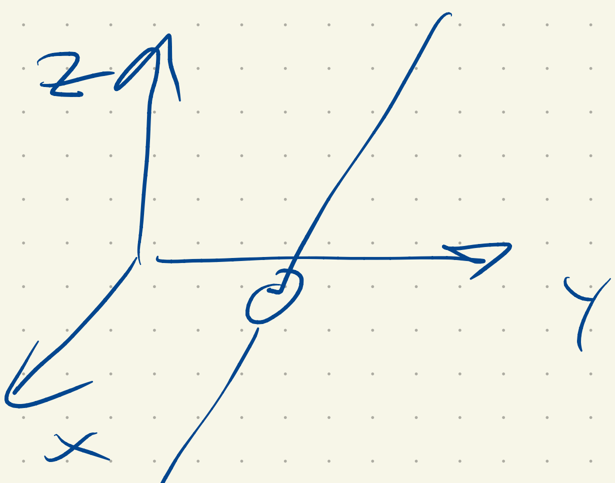
$$\vec{r} = \langle 3 + 2t, -1 + 6t, 7 + t \rangle$$

where does this line intersect the $\underbrace{\text{x-y plane}}$

$\hookrightarrow z = 0$



$$z=0 \Rightarrow 7+t=0$$

$$t=-7$$

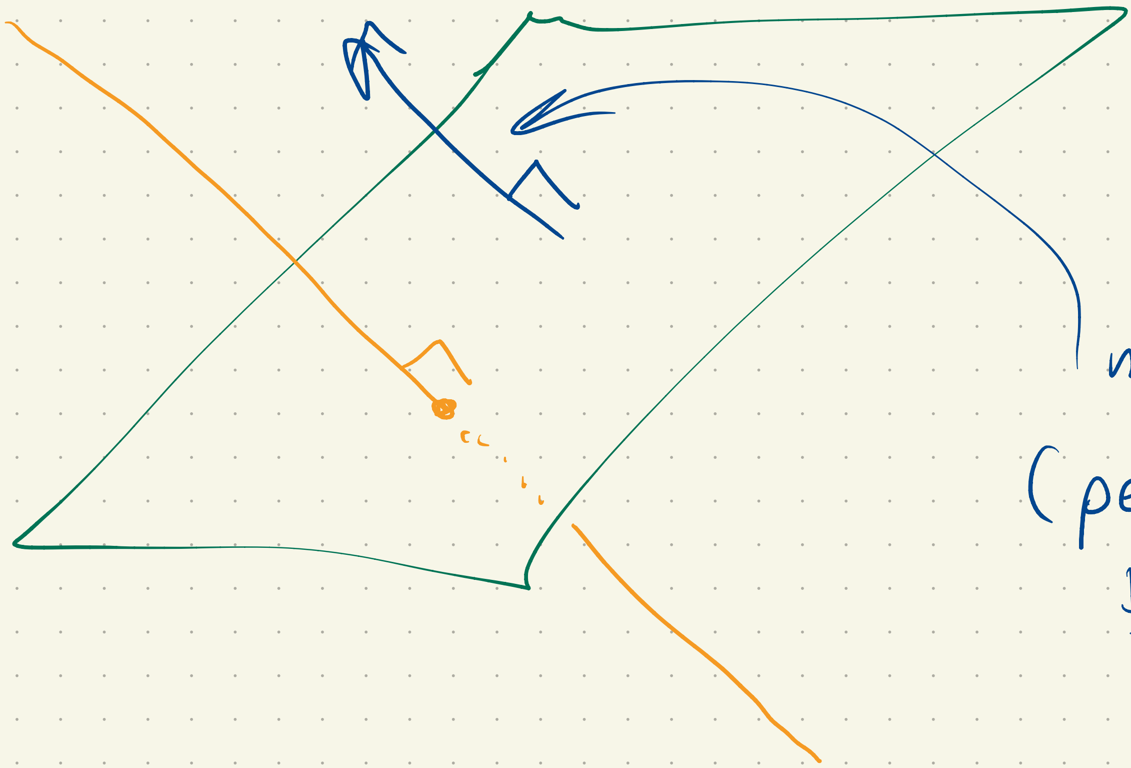
$$x = 3+2\cdot(-7) = -11$$

$$y = -1+6\cdot(-7) = -43$$

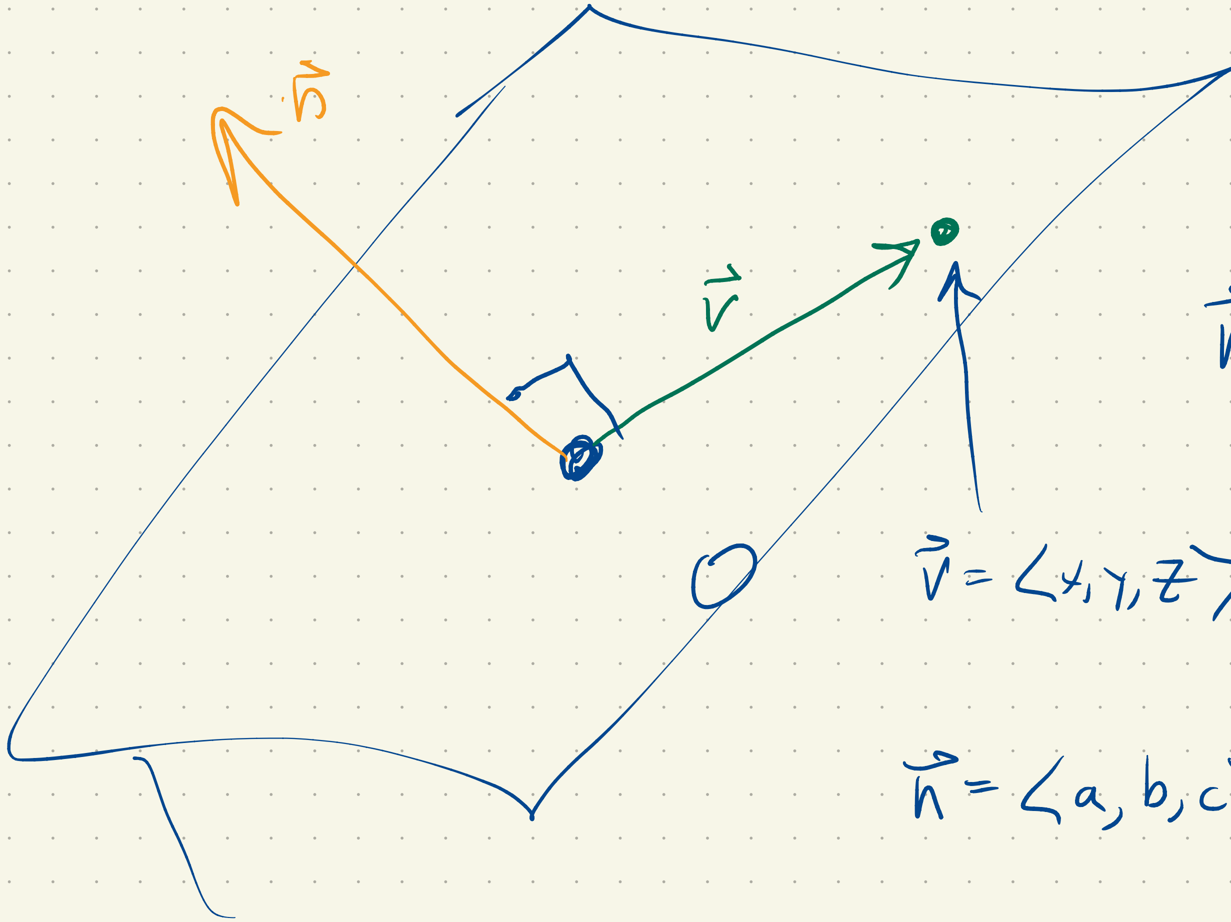
$$\langle -11, -43, 0 \rangle \leftarrow$$

$$(-11, -43, 0)$$

Equations of planes



normal vector (perpendicular to the directions parallel to the plane)



$$\vec{n} \cdot \vec{v} = 0$$

$\vec{v} = \langle x, y, z \rangle$ $\quad P(x, y, z)$

$\vec{n} = \langle a, b, c \rangle$

$\vec{n} \cdot \vec{v} = 0 \rightarrow$

$$ax + by + cz = 0$$

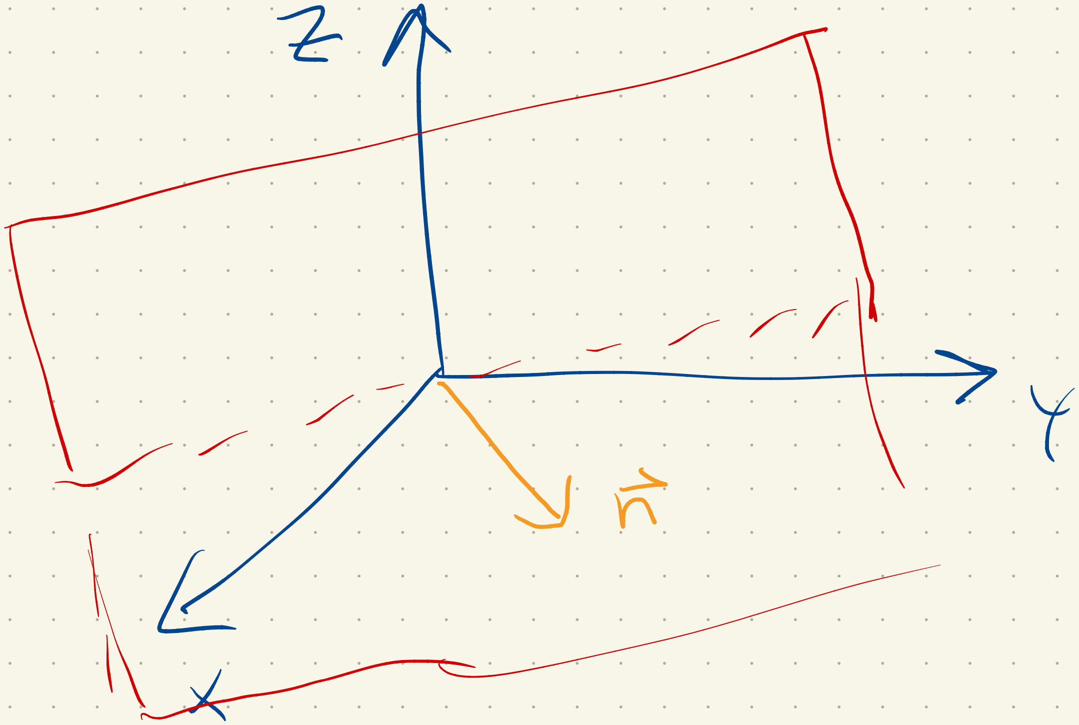
Every plane through the origin has this form

$$6x - 2y + 7z = 0$$

$\langle 6, -2, 7 \rangle = \vec{n}$ normal vector

Given $\vec{n} = \langle 1, 1, 0 \rangle$ is $P(3, 2, 6)$ on the plane thru O with this normal vector?

Is $Q(-1, 1, 5)$



$1\cdot x + 1\cdot y + 0\cdot z = 0$

$1\cdot 3 + 1\cdot 2 + 0\cdot 6 = 5$

$\neq 0$

So P is not in the plane.

$\vec{n} = \langle 1, 1, 0 \rangle$

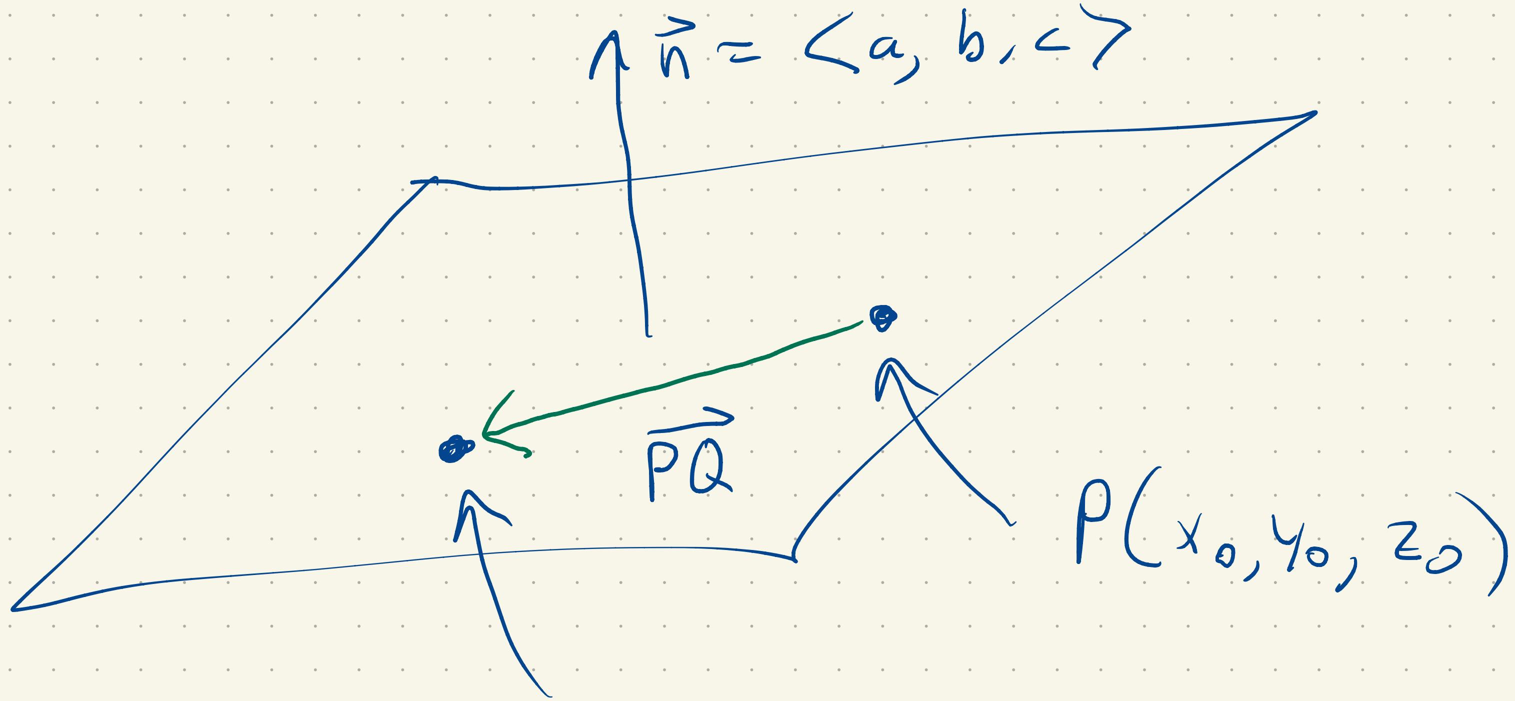
$\langle 0, 0, z \rangle$

$1\cdot 0 + 1\cdot 0 + 0\cdot z = 0$

$1\cdot(-1) + 1\cdot 1 + 0\cdot 5$

$= -1 + 1 = 0$

So Q is in this plane.



$Q(x, y, z)$

$$\vec{PQ} = \langle x - x_0, y - y_0, z - z_0 \rangle$$

$$\vec{n} \cdot \vec{PQ} = 0$$

$$\downarrow$$

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

$$ax + by + cz = 0$$

$$6(x-1)+2(y-7)+3(z+9)=0$$

$$\underbrace{\langle 6, 2, 3\rangle}_{\vec{n}} \qquad \underbrace{\langle 1, 7, -9\rangle}$$

same point on the plane

$$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$$

$$ax+by+cz=\underbrace{ax_0+by_0+cz_0}_{d}$$

$$ax + by + cz = d$$

a, b, c, d are numbers

$$6(x-1) + 2(y-7) + 3(z+9) = 0$$

$$6x - 6 + 2y - 14 + 3z + 27 = 0$$

$$6x + 2y + 3z - 20 + 27 = 0$$

$$6x + 2y + 3z = -7$$

$$6x + 2y + 3z = 0$$

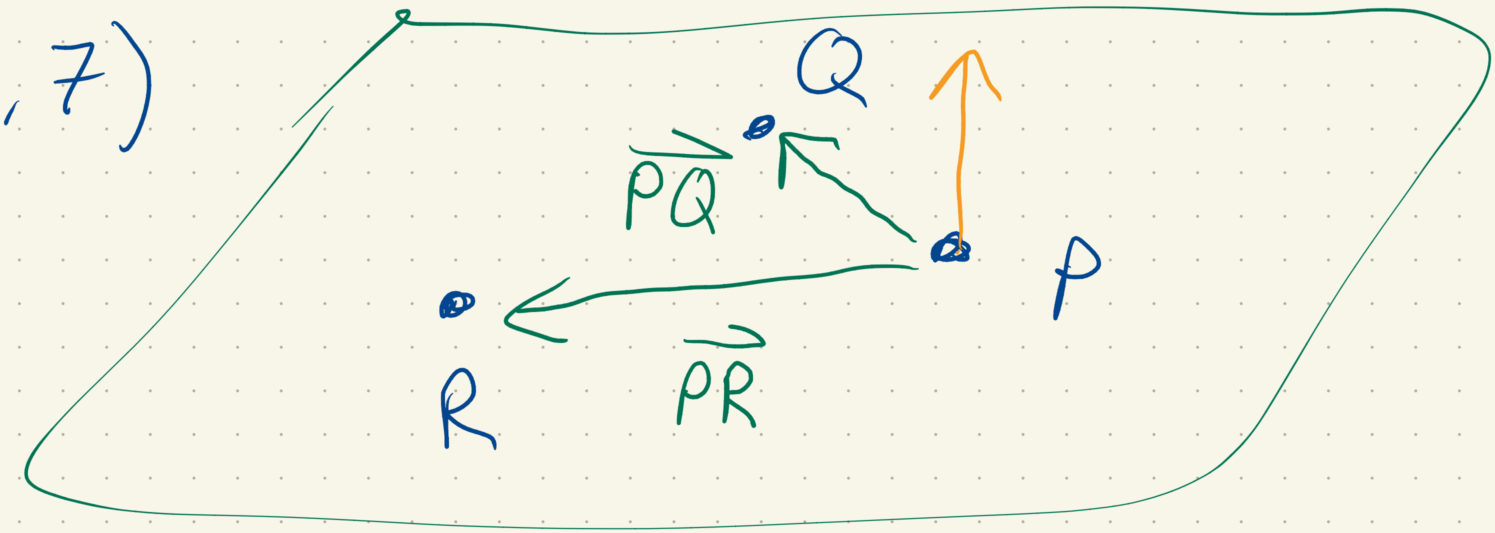
plane thru origin

$P(1,0,2)$

$Q(-1,3,4)$

$R(3,5,7)$

Find plane passing through these three points.



$$\vec{n} = \overrightarrow{PQ} \times \overrightarrow{PR}$$

$$\overrightarrow{PQ} = \langle -2, 3, 2 \rangle$$

$$\overrightarrow{PR} = \langle 2, 5, 5 \rangle$$

$$\begin{vmatrix} \hat{i} & \hat{j} & k \\ -2 & 3 & 2 \\ 2 & 5 & 5 \end{vmatrix} = \hat{i}\,(3\cdot 5 - 2\cdot 5)$$

$$- \hat{j}\,(-2\cdot 5 - 2\cdot 2)$$

$$+ \hat{k}\,(-2\cdot 5 - 3\cdot 2)$$

$$= \hat{i}\cdot 5 - \hat{j}\cdot(-14) + \hat{k}\cdot(-16)$$

$$= 5\hat{i} + 14\hat{j} - 16\hat{k}$$

$$\vec{n} = \langle 5, 14, -16 \rangle$$

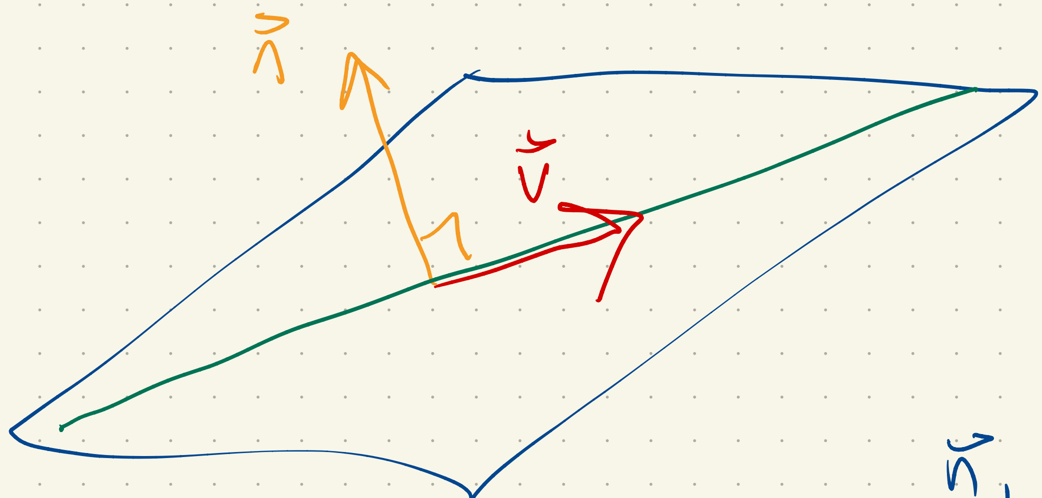
$P(1,0,2)$

$$5(x-1)+14(y-0)-16(z-2)=0$$

Given $x+y+z=1$ what's the line of intersection?

$x-2y+3z=1$

direction perpendicular to both normal vectors



$\vec{n}_1=\langle 1,1,1\rangle$

$\vec{n}_2=\langle 1,-2,3\rangle$

$$\vec{v} = \vec{n}_1 \times \vec{n}_2$$

$$= \langle 5, -2, -1 \rangle$$

We need a single point on the line.