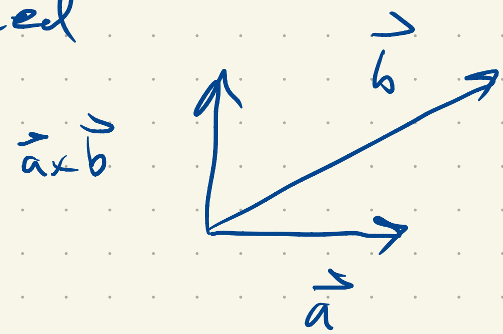
$\vec{a} \times \vec{b} \perp \vec{a}, \perp \vec{b}$

points in a direction so that $(\vec{a}, \vec{b}, \vec{a} \times \vec{b})$ is
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad 1 \quad 2 \quad 3$

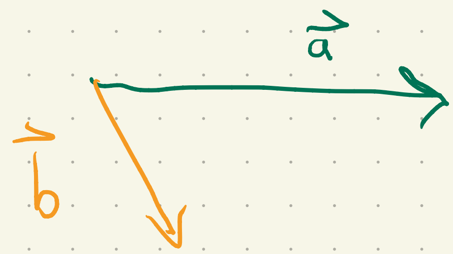
right handed

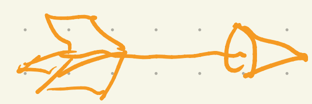



points at you

$\odot$ $\otimes$

points away



$$\|\vec{a} \times \vec{b}\| = \|\vec{a}\| \|\vec{b}\| \sin \theta$$


area $\|\vec{a}\|\,\|\vec{b}\|\sin\theta$

$0 \le \theta \le \pi$

$\vec{a}\cdot\vec{b} = 0 \iff \vec{a} \perp \vec{b}$

$\vec{a}\times\vec{b} = \vec{0}$

$\|\vec{a}\times\vec{b}\| = 0$

$\|\vec{a}\|\,\|\vec{b}\|\sin\theta = 0$

$\sin\theta = 0$

$\Rightarrow \theta = 0$ or $\pi$

$\vec{a}\times\vec{b} = \vec{0} \iff \vec{a}$ and $\vec{b}$ are parallel.

$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$$

$$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$$

$$(k\vec{a}) \times \vec{b} = k(\vec{a} \times \vec{b})$$

$$(\vec{a} \times \vec{b}) \times \vec{c} \overset{?}{=} \vec{a} \times (\vec{b} \times \vec{c})$$

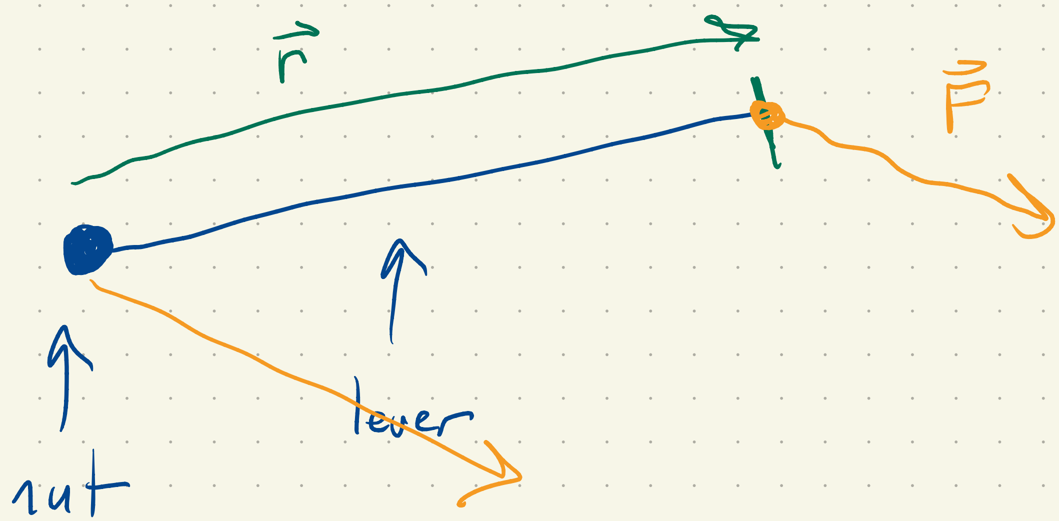
$$(\hat{i} \times \hat{i}) \times \hat{j} \qquad\qquad \hat{i} \times (\hat{i} \times \hat{j})$$

$$\vec{0} \times \hat{j} \qquad\qquad \hat{i} \times \hat{k}$$

$$\vec{0} \qquad\qquad -\hat{j}$$

$$\hat{i} \quad \hat{j} \quad \hat{k} \quad \hat{i} \quad \hat{j}$$

# Application Torque

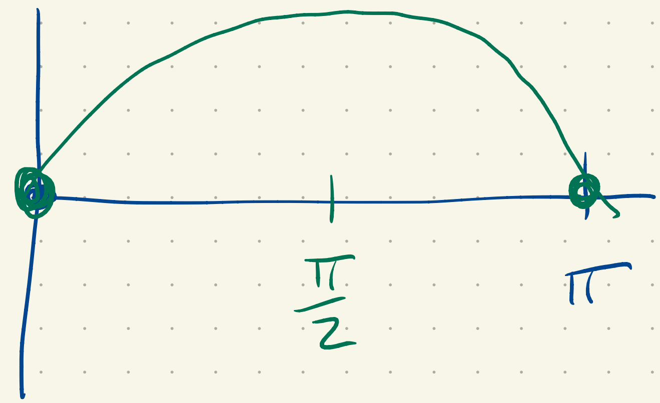


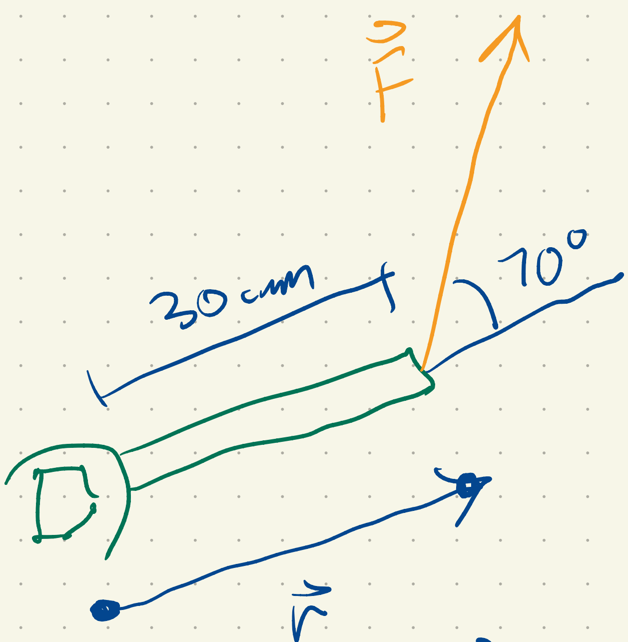

By def: the applied torque is $\vec{\tau} = \vec{r} \times \vec{F}$

$$[\vec{\tau}] = \begin{matrix} ft \cdot lbs \\ m \; N \end{matrix}$$

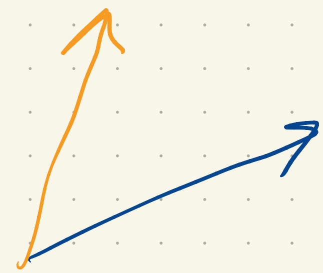
$0 \leq \theta \leq \pi$

$$\|\vec{\tau}\| = \|\vec{r} \times \vec{F}\| = \|\vec{r}\| \, \|\vec{F}\| \sin \measuredangle\theta$$

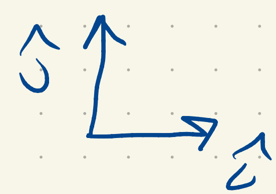



$\|\vec{F}\| = 100\ N$



What is the resulting torque?




$\vec{\tau} = \vec{r} \times \vec{F}$

$\odot$

$$\|\vec{\tau}\| = \|\vec{r}\|\,\|\vec{F}\| \sin\theta \qquad (\hat{\imath}, \hat{\jmath}, \hat{k})$$

$$= 0.3 \cdot 100 \sin 70^\circ$$

$$= 28.2 \text{ Nm}$$

$$\vec{\tau} = 28.2\,\hat{k} \text{ Nm}$$

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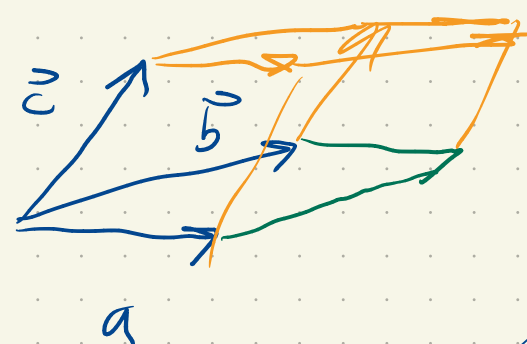
scalar triple product

$$\vec{c}\cdot(\vec{a}\times\vec{b})$$

parallel piped




volume of spanned

up to sign. positive if $(\hat{a}, \vec{b}, \vec{c})$ is right handed.

# Section 2.5 (lines + planes in space)

2d $\qquad y = mx + b$

$$y - y_0 = m(x - x_0) \quad \text{(point slope)}$$

(problem w/ vertical lines)

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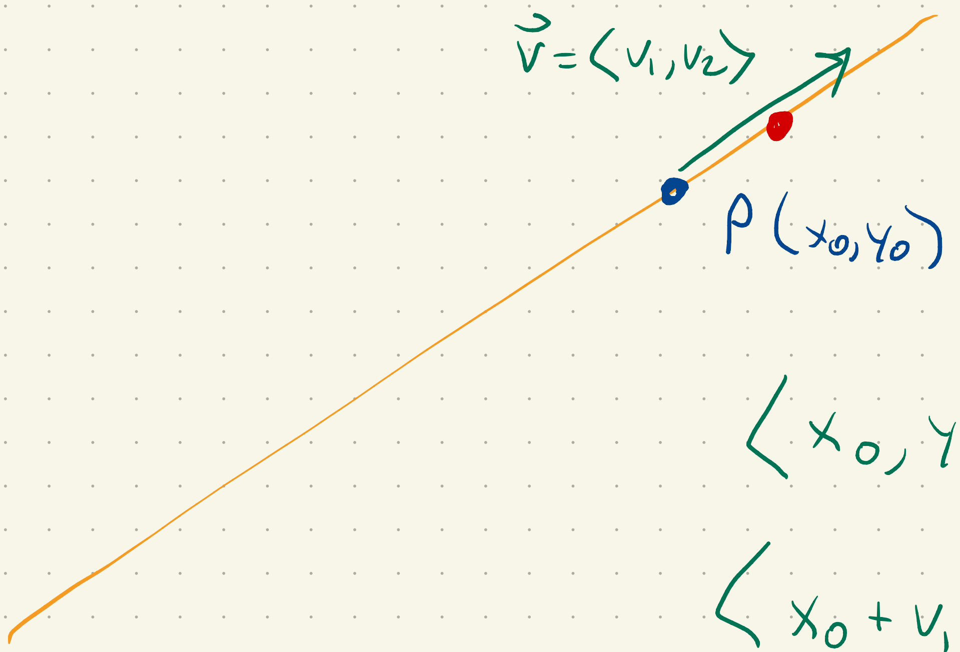



$$\langle x_0, y_0 \rangle$$

$$\langle x_0 + v_1, y_0 + v_2 \rangle = \langle x_0, y_0 \rangle + \vec{v}$$

$$\langle x_0, y_0 \rangle + \frac{1}{2}\vec{v}$$

# Vector form of a line

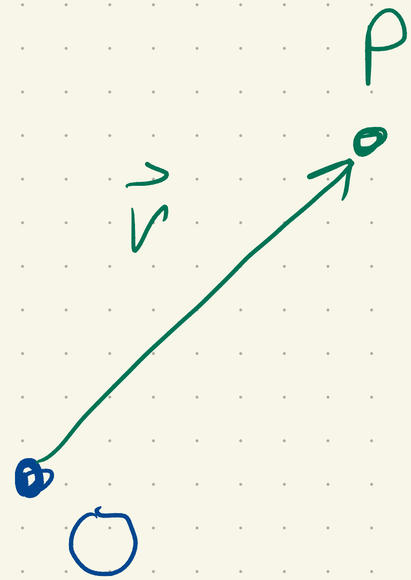
$$\vec{r} = \vec{r}_0 + t\vec{v}$$

$\vec{r}_0 = \langle x_0, y_0 \rangle$

$\vec{v} = \langle v_1, v_2 \rangle$

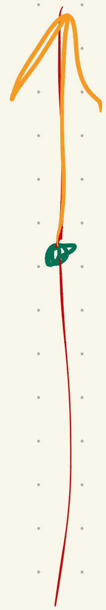
$(x_0, y_0)$

$\langle x_0, y_0 \rangle$




$\vec{v} = \langle v_1, v_2 \rangle$

$P(v_1, v_2)$



$(2,3) = \vec{r}_0$

$\vec{v} = \langle 0, 3 \rangle$

With $\vec{v} = \langle v_1, v_2 \rangle$ we can determine the slope of the line

$\vec{r}_0 = \langle x_0, y_0 \rangle$

$\vec{v} = \langle v_1, v_2 \rangle$

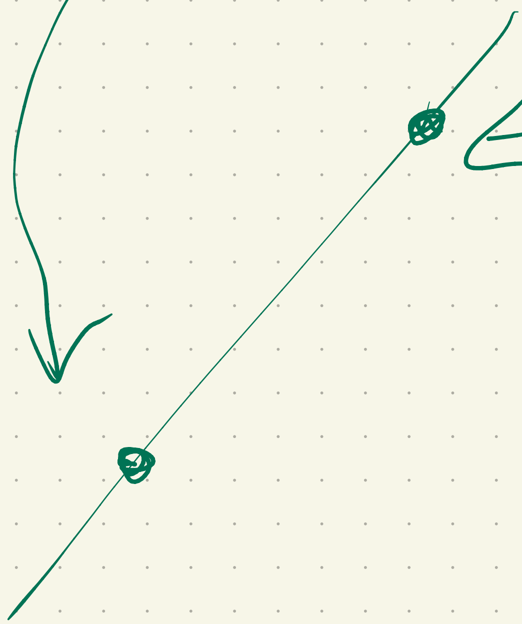
$$\vec{r} = \vec{r}_0 + t\vec{v}$$

Two points on it:

a) $t = 0$ $\quad \vec{r}_0 = \langle x_0, y_0 \rangle$

b) $t = 1$ $\quad \vec{r} = \langle x_0 + v_1, y_0 + v_2 \rangle$

$\langle x_0, y_0 \rangle$ $\langle x_0 + v_1, y_0 + v_2 \rangle$



$\Delta x = v_1$

$\Delta y = v_2$

Slope: $\dfrac{v_2}{v_1}$