Last class

$$\vec{a} = \langle a_1, a_2, a_3 \rangle$$

$$\vec{b} = \langle b_1, b_2, b_3 \rangle$$

$$\vec{a} \times \vec{b} = \langle a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1 \rangle$$

$\uparrow$

cross product

$-5 \cdot 7$

$$\vec{a} \cdot (\vec{a} \times \vec{b}) = 0$$

$$\vec{b} \cdot (\vec{a} \times \vec{b}) = 0$$

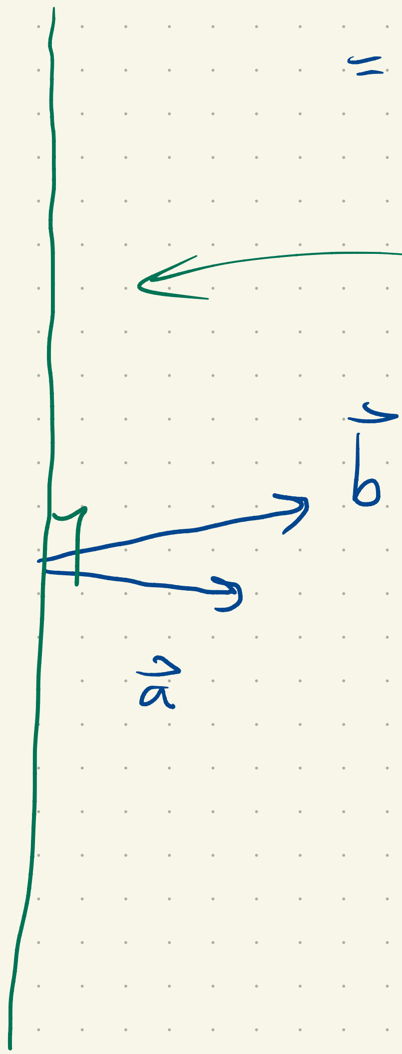
$$\vec{b} \times \vec{a} = \langle b_2 a_3 - b_3 a_2, \text{---}, \text{---} \rangle = -\vec{a} \times \vec{b}$$

$$\vec{a} \cdot (\vec{a} \times \vec{b}) = 0$$

$$\vec{b} \cdot (\vec{a} \times \vec{b}) = \vec{b} \cdot \left(-\vec{b} \times \vec{a}\right)$$

$$= -\left(\vec{b} \cdot (\vec{b} \times \vec{a})\right) = -0$$

$$= 0$$



$\vec{a} \times \vec{b}$ lives here

How to compute (3x3 determinants)

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \vec{a} \times \vec{b}$$

$$\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & \boxed{\begin{matrix} a_2 & a_3 \\ b_2 & b_3 \end{matrix}} \\ b_1 & \end{vmatrix} \rightarrow +\hat{i} \cdot \left(a_2 b_3 - a_3 b_2 \right)$$

$$+\hat{j}\,(-a_1 b_3 + a_3 b_1)$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \boxed{a_1} & a_2 & \boxed{a_3} \\ \boxed{b_1} & b_2 & \boxed{b_3} \end{vmatrix} \rightarrow -\hat{j}\,(a_1 b_3 - a_3 b_1)$$

$$\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \rightarrow +\hat{k}\,(a_1 b_2 - a_2 b_1)$$

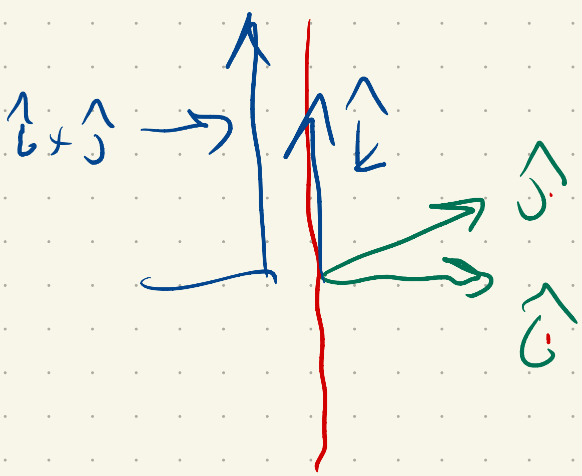
$\hat{i} \times \hat{j}$

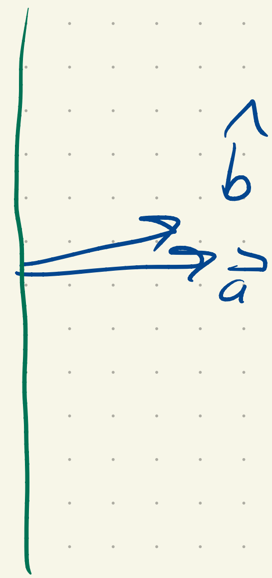
$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = \hat{i} \cdot 0 - \hat{j} \cdot 0 + \hat{k}\left(1 \cdot 1 - 0 \cdot 0\right)$$

$$= \hat{k}$$

$\hat{i} = \langle 1, 0, 0 \rangle$

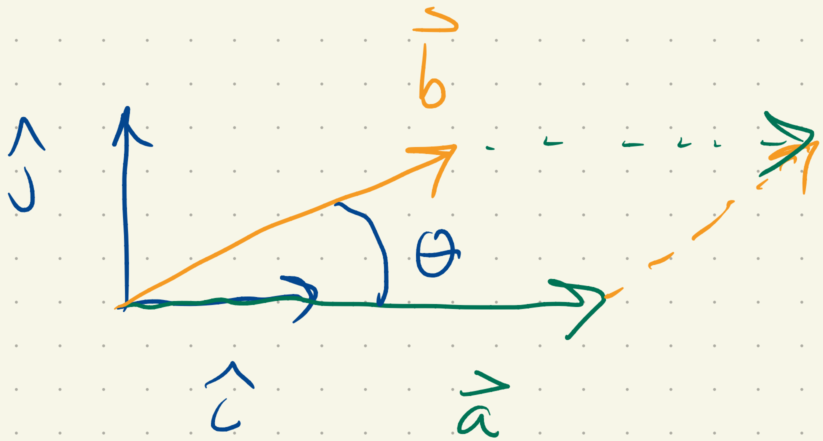
$\hat{j} = \langle 0, 1, 0 \rangle$





$\hat{j} \times \hat{i} = -\hat{k}$

How long is $\vec{a} \times \vec{b}$?



$0 \leq \theta \leq \pi$

$$\vec{a} = \|\vec{a}\| \hat{i}$$

$$\vec{b} = \|\vec{b}\| \cos\theta \hat{i} + \|\vec{b}\| \sin\theta \hat{j}$$

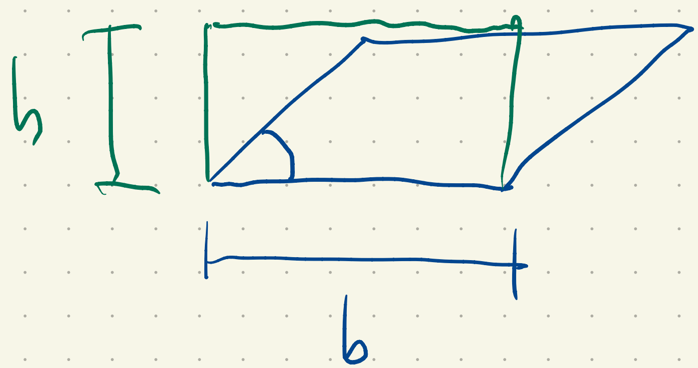
$$\vec{a} \times \vec{b} = \|\vec{a}\| \hat{i} \times \left(\|\vec{b}\| \cos\theta \hat{i} + \|\vec{b}\| \sin\theta \hat{j} \right)$$

$$= \|\vec{a}\| \|\vec{b}\| \cos\theta \underbrace{\hat{\imath} \times \hat{\imath}}_{\vec{0}} + \|\vec{a}\| \|\vec{b}\| \sin\theta \; \hat{\imath} \times \hat{\jmath}$$

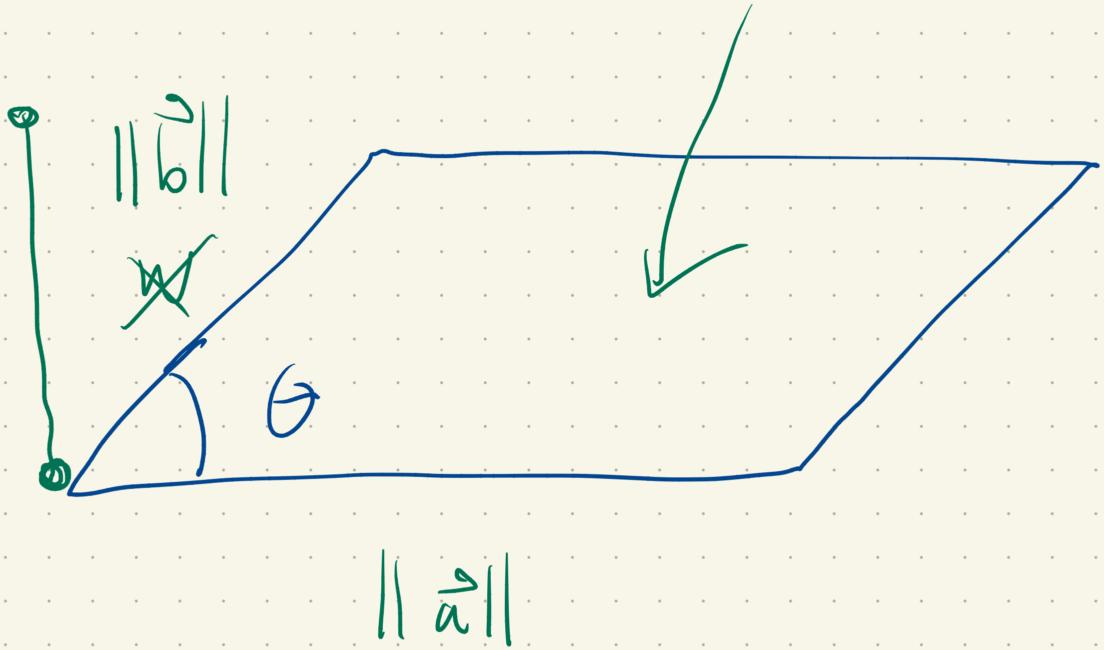
$$\vec{a} \times \vec{a} = \vec{0}$$

$$= \|\vec{a}\| \|\vec{b}\| \sin\theta \; \hat{k}$$

$$\|\vec{a} \times \vec{b}\| = \|\vec{a}\| \|\vec{b}\| \sin\theta \qquad 0 \leq \theta \leq \pi$$



area $b \cdot h$



$\cancel{w \sin\theta}$

$\|\vec{b}\| \sin\theta$

Area of parallelogram $\|\vec{a}\|\,\|\vec{b}\| \sin\theta$

$$\downarrow$$

$$\|\vec{a} \times \vec{b}\|$$