

Last class

$$\vec{a} = \langle a_1, a_2, a_3 \rangle$$

$$\vec{b} = \langle b_1, b_2, b_3 \rangle$$

$$\vec{a} \times \vec{b} = \langle a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1 \rangle$$

↑
cross product

- 5.7

$$\vec{a} \cdot (\vec{a} \times \vec{b}) = 0$$

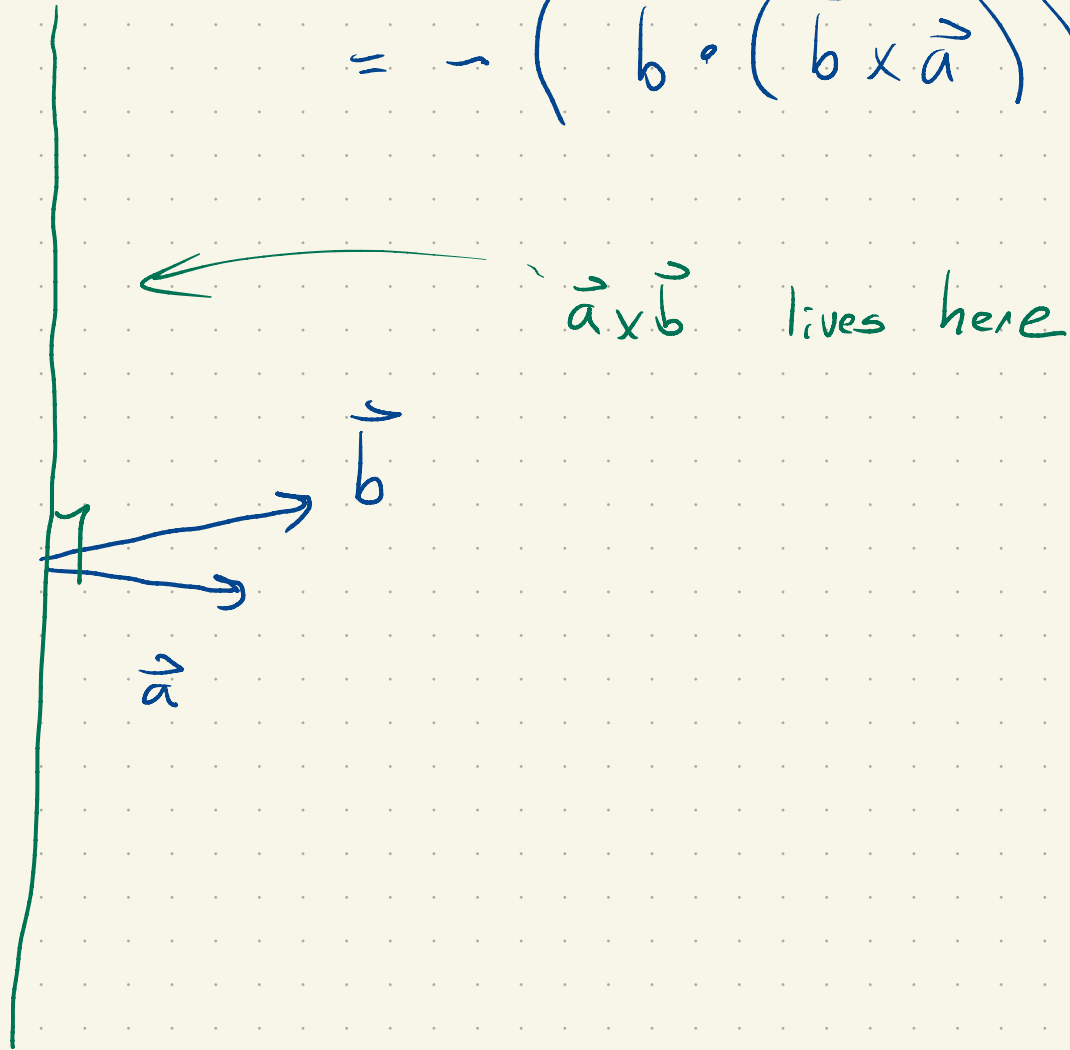
$$\vec{b} \cdot (\vec{a} \times \vec{b}) = 0$$

$$\vec{b} \times \vec{a} = \langle b_2 a_3 - b_3 a_2, \quad \quad \quad \rangle = -\vec{a} \times \vec{b}$$

$$\vec{a} \cdot (\vec{a} \times \vec{b}) = 0$$

$$\vec{b} \cdot (\vec{a} \times \vec{b}) = \vec{b} \cdot (-\vec{b} \times \vec{a})$$

$$= -(\vec{b} \cdot (\vec{b} \times \vec{a})) = -0 = 0$$



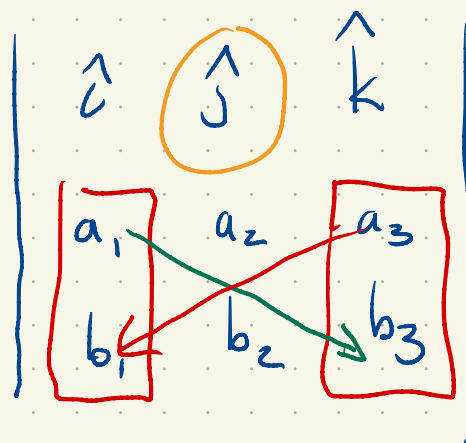
How to compute

(3x3 determinants)

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \vec{a} \times \vec{b}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \rightarrow +\hat{i} \cdot \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - \hat{j} \cdot \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + \hat{k} \cdot \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

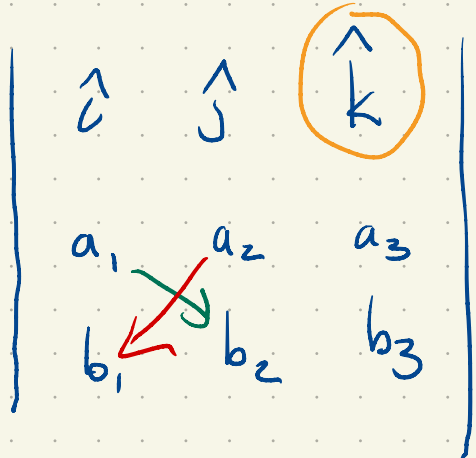
The diagram shows the expansion of the determinant along the first row. The first element \hat{i} is circled in orange. A red box highlights the 2x2 sub-determinant $\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix}$ with a red arrow pointing to the $+\hat{i}$ term. A green arrow points from a_2 to b_3 and a red arrow points from a_3 to b_2 within the sub-determinant.



$$+ \hat{j} (-a_1 b_3 + a_3 b_1)$$

$$\Rightarrow - \hat{j} (a_1 b_3 - a_3 b_1)$$

$$\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix}$$

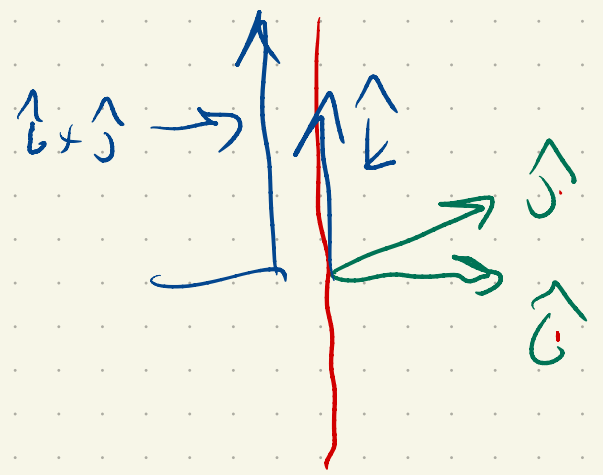
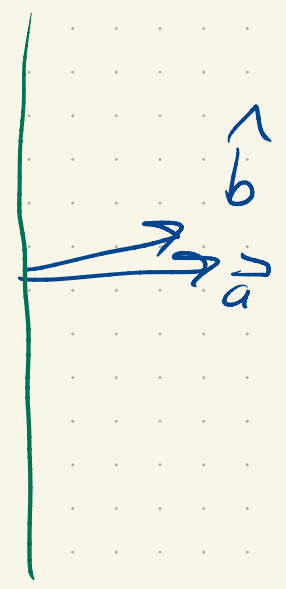


$$\Rightarrow + \hat{k} (a_1 b_2 - a_2 b_1)$$

$$\hat{i} \times \hat{j} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = \hat{i} \cdot 0 - \hat{j} \cdot 0 + \hat{k} (1 \cdot 1 - 0 \cdot 0) = \hat{k}$$

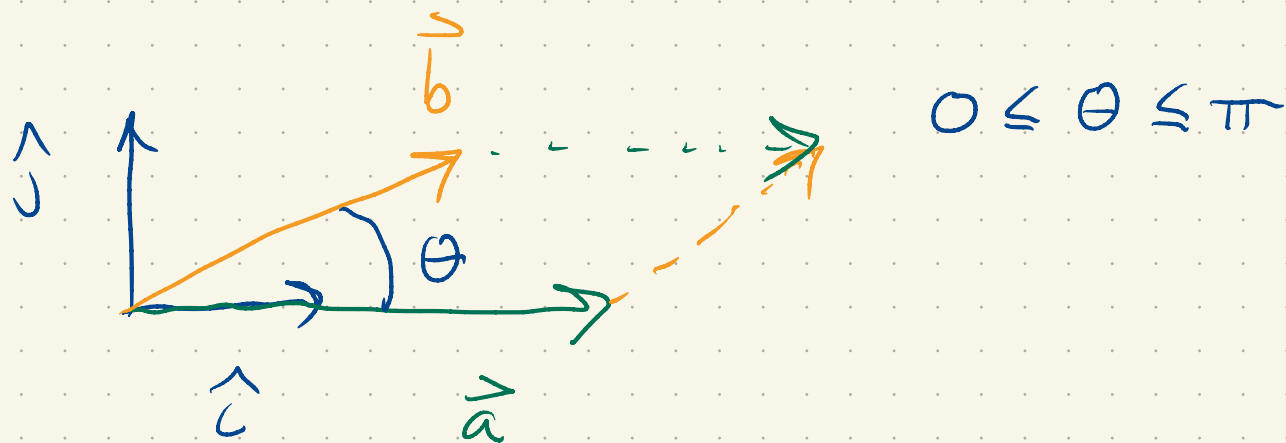
$$\hat{i} = \langle 1, 0, 0 \rangle$$

$$\hat{j} = \langle 0, 1, 0 \rangle$$



$$\hat{j} \times \hat{i} = -\hat{k}$$

How long is $\vec{a} \times \vec{b}$?



$$\vec{a} = \|\vec{a}\| \hat{i}$$

$$\vec{b} = \|\vec{b}\| \cos \theta \hat{i} + \|\vec{b}\| \sin \theta \hat{j}$$

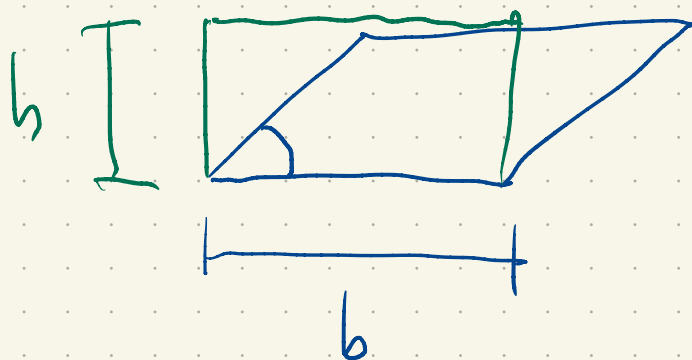
$$\vec{a} \times \vec{b} = \|\vec{a}\| \hat{i} \times \left(\|\vec{b}\| \cos \theta \hat{i} + \|\vec{b}\| \sin \theta \hat{j} \right)$$

$$= \|\vec{a}\| \|\vec{b}\| \cos\theta \underbrace{\vec{a} \times \vec{a}}_{\vec{0}} + \|\vec{a}\| \|\vec{b}\| \sin\theta \vec{a} \times \hat{b}$$

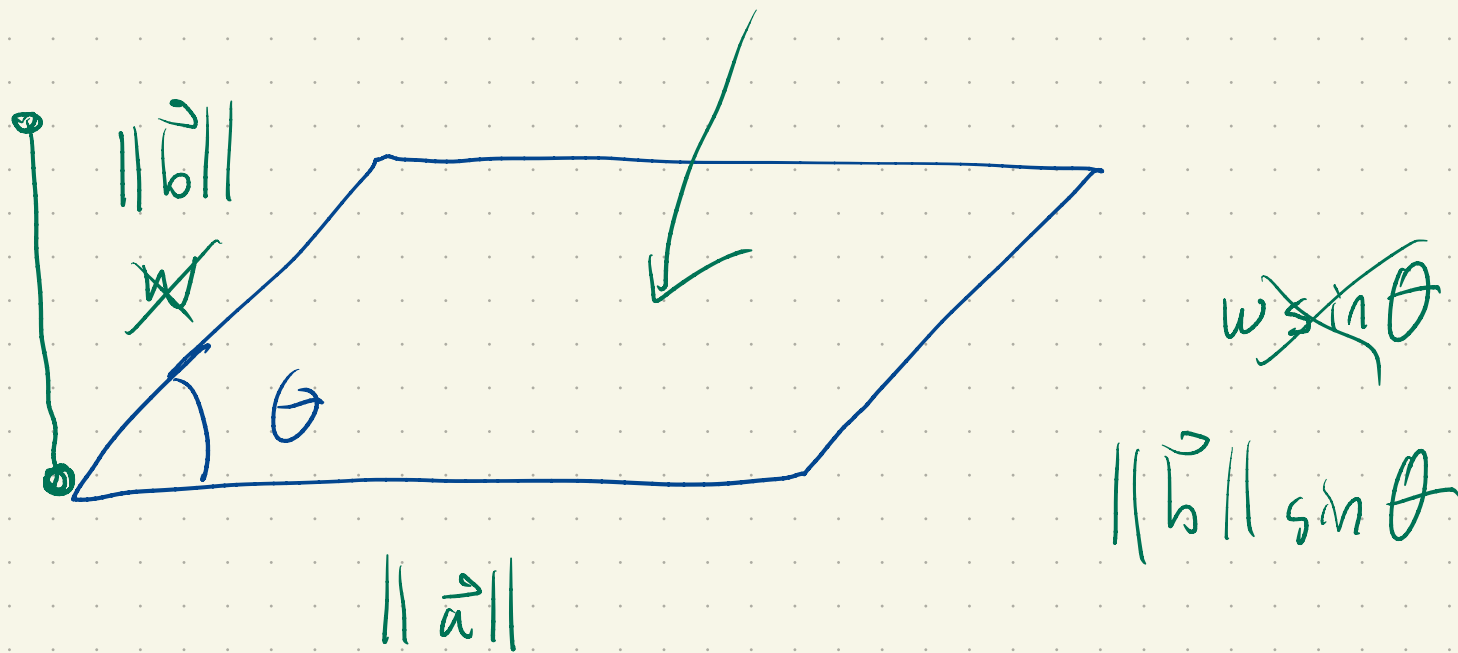
$\vec{a} \times \vec{a} = \vec{0}$

$$= \|\vec{a}\| \|\vec{b}\| \sin\theta \hat{k}$$

$$\|\vec{a} \times \vec{b}\| = \|\vec{a}\| \|\vec{b}\| \sin\theta \quad 0 \leq \theta \leq \pi$$



area $b \cdot h$



Area of parallelogram $\|\vec{a}\| \|\vec{b}\| \sin \theta$

\downarrow

$\|\vec{a} \times \vec{b}\|$