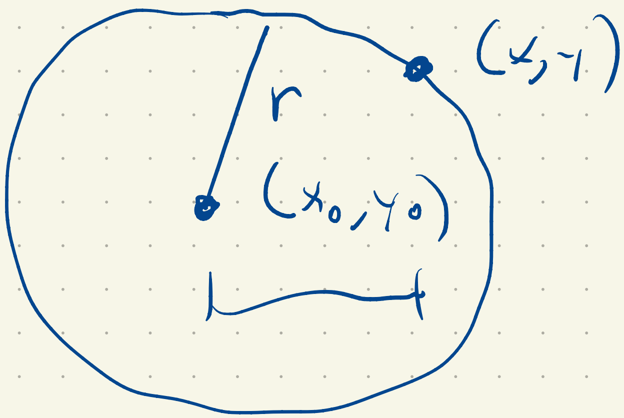
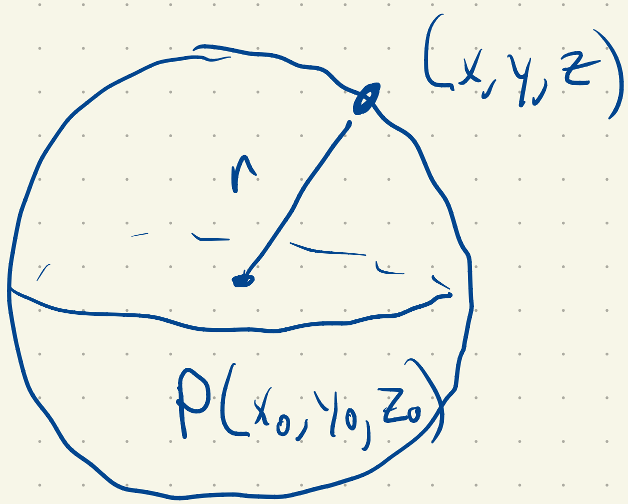


$$r^2 = (x - x_0)^2 + (y - y_0)^2$$

$$x^2 + y^2 + z^2 = r^2$$



$$(x - x_0)^2 + (y - y_0) + (z - z_0)^2 = r^2$$

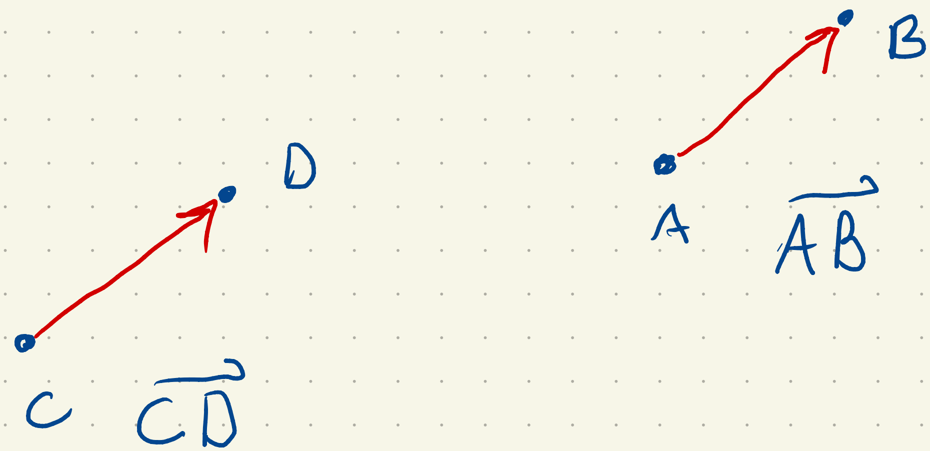
$$\Delta x^2 + \Delta y^2 + \Delta z^2 = r^2$$

$$\downarrow$$

$$\Delta x = x - x_0$$

Vectors

Displacement Vectors



$$\overrightarrow{CD} = \overrightarrow{AB} \qquad C \neq A$$

$$D \neq B$$

displacement vector

Displacement vectors (mostly) have a length and a direction.

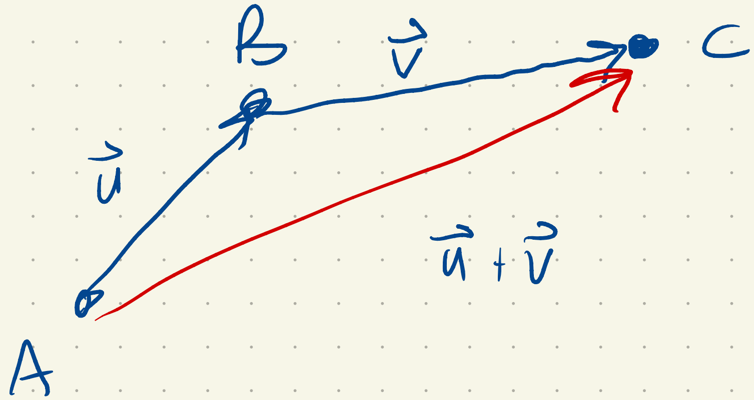
Length: $|\overrightarrow{AB}|$ = distance from A to B.

The vector with length zero (zero vector) does not have a direction

Operations with vectors:

1) Vector addition

$\vec{u} + \vec{v}$



$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$

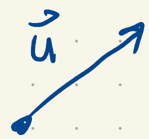
2) Scalar multiplication

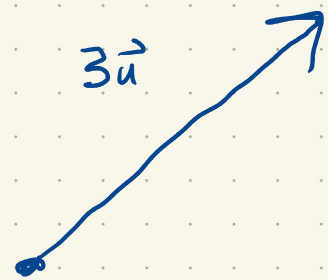
a, real number

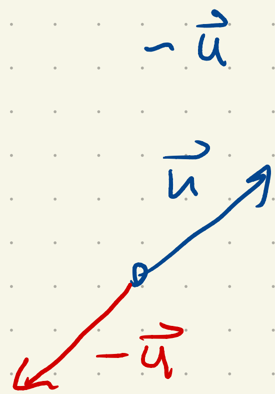
$\vec{u}$, vector, $\vec{u} \neq \vec{0}$ ← zero vector

$a\vec{u}$ is the vector parallel to $\vec{u}$ (pointing in same direction)

with length $a \cdot |\vec{u}|$







$a < 0$ $\qquad a\vec{u} = ?$

has length $|a|\,|\vec{u}|$

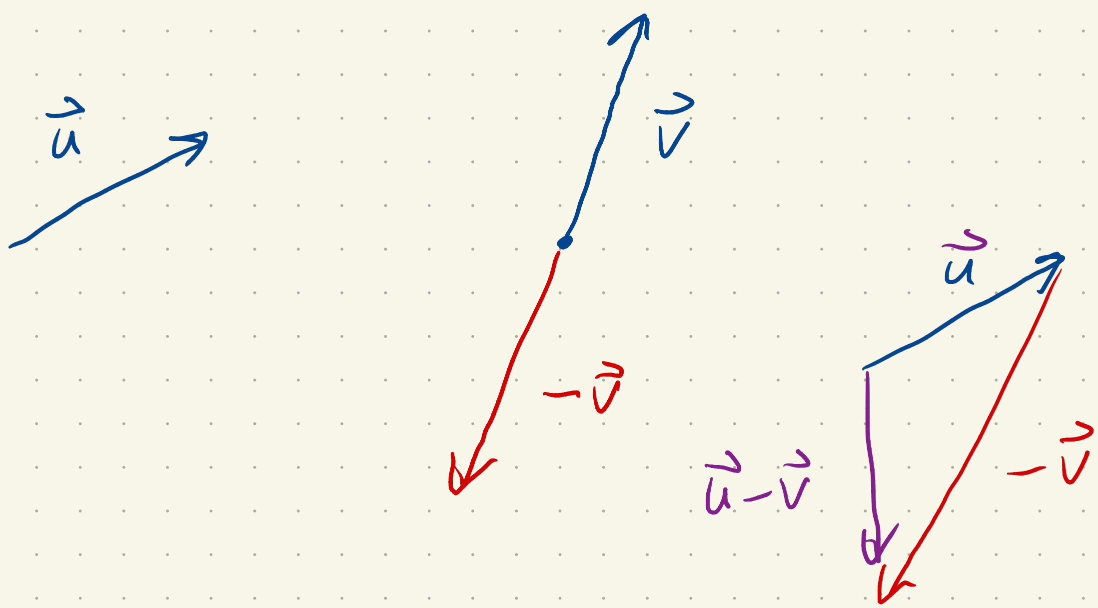
and points in opposite direction

$a = 0$ $\qquad a\vec{u} = \vec{0}$ $\qquad$ | a anything

$a\vec{0} = \vec{0}$

Subtraction

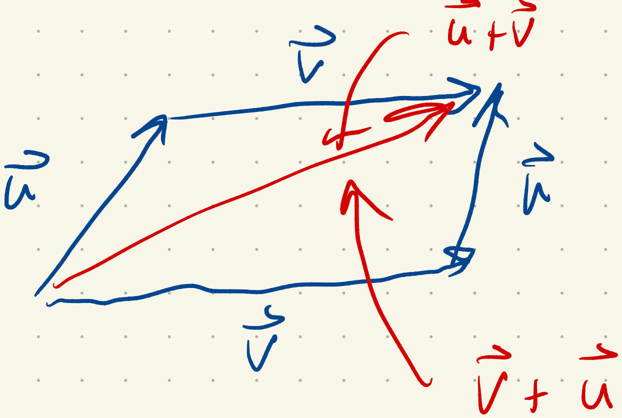
$$\vec{u} - \vec{v} = \vec{u} + (-\vec{v})$$



$\vec{u} - \vec{v}$

$$\vec{u} + \vec{v} \stackrel{?}{=} \vec{v} + \vec{u}$$

$\vec{u}$



Zero vector is special:

$$\vec{0}+\vec{u}=\vec{u}=\vec{u}+\vec{0}$$

Origin $(0,0,0)$

When you have cartesian coordinates

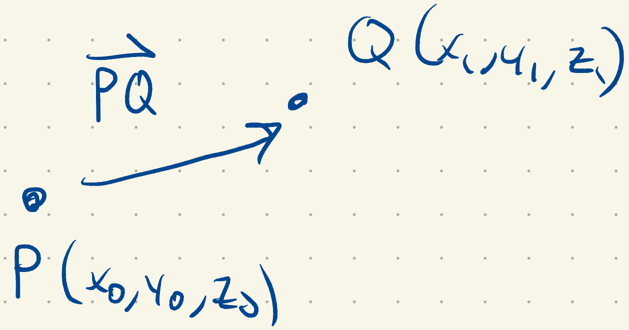
then vectors also inherit coordinates.

$P(x_0, y_0, z_0)$

$Q(x_1, y_1, z_1)$

$$\overrightarrow{PQ} = \langle x_1 - x_0,\ y_1 - y_0,\ z_1 - z_0 \rangle$$

difference in the coordinates of P and Q



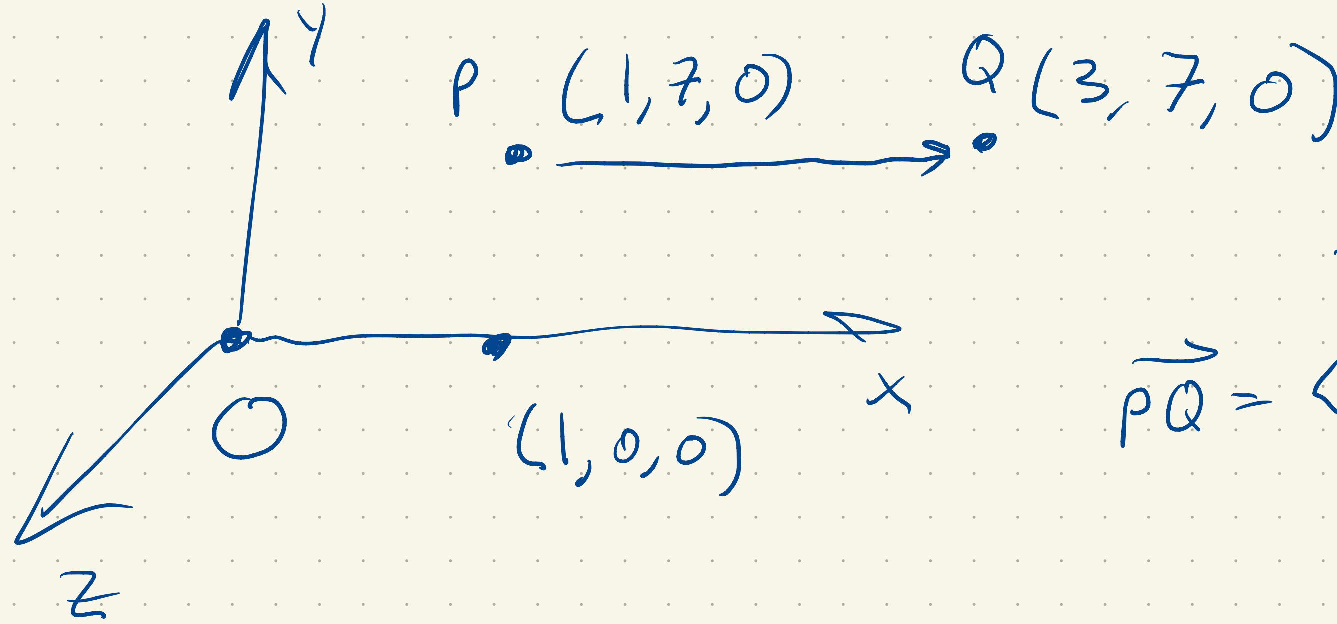
Operations:

components

$$\vec{a} = \langle a_1, a_2, a_3 \rangle$$

$$\vec{b} = \langle b_1, b_2, b_3 \rangle$$

$$\vec{a} + \vec{b} = \langle a_1 + b_1,\ a_2 + b_2,\ a_3 + b_3 \rangle$$



$3-1 \quad 7-7 \quad 0-0$

$$\overrightarrow{PQ} = \langle 2, 0, 0 \rangle$$

$$\vec{a} = \langle a_1, a_2, a_3 \rangle \quad , \quad \vec{b} = \langle b_1, b_2, b_3 \rangle$$

$$c\,\vec{a} = \langle ca_1, ca_2, ca_3 \rangle$$

$$\vec{a} - \vec{b} = \langle a_1 - b_1, a_2 - b_2, a_3 - b_3 \rangle$$

Properties:

$$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$$

$$c(\vec{a} + \vec{b}) = c\vec{a} + c\vec{b}$$

$\vec{a} + \vec{0} = \vec{a}$ | $1\,\vec{a} = \vec{a}$

The length of a vector is the Euclidean length of the displacement.

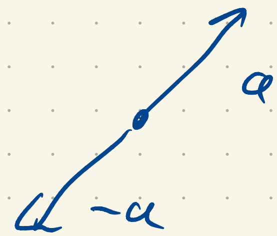
$$\vec{a} = \langle a_1, a_2, a_3 \rangle$$

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

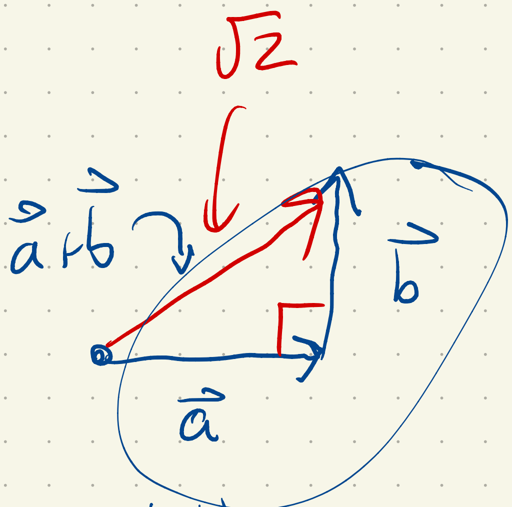
$\vec{a} = \overrightarrow{PQ}$

$|\vec{a}|$ is the dist. from $\vec{P}$ to $\vec{Q}$.

$$|c\vec{a}| = |c||\vec{a}|$$



$$|\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$$ ← Triangle inequality

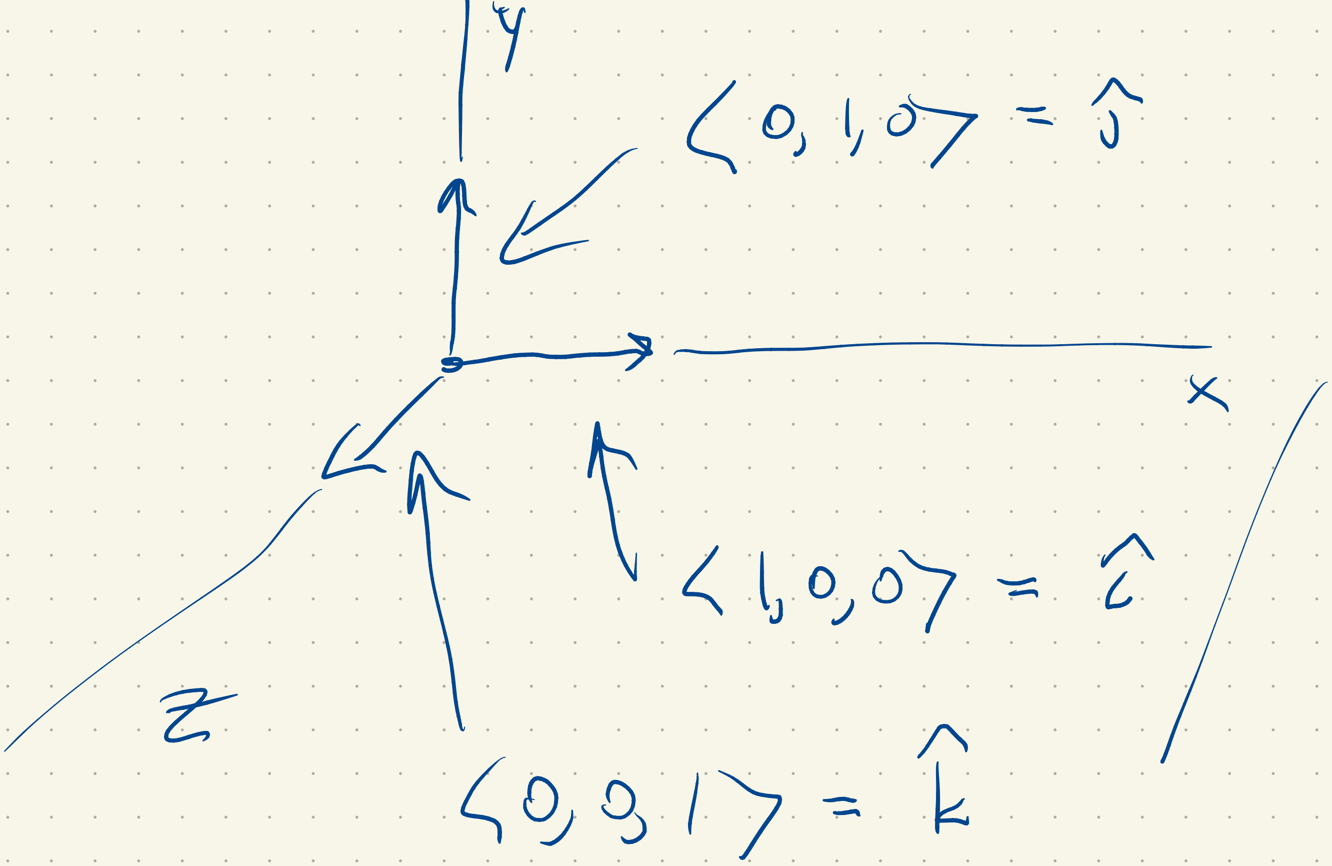


$|\vec{b}| = 1$

$|\vec{a}| + |\vec{b}| = |\vec{a}+\vec{b}|$

$|\vec{a}| = 1$

$$\vec{a} + (-\vec{a}) = \vec{0}$$



$\langle 0,1,0\rangle = \hat{j}$

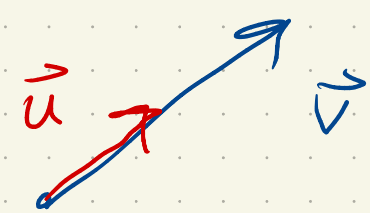
$\langle 1,0,0\rangle = \hat{i}$

$\langle 0,0,1\rangle = \hat{k}$

$\vec{v}$

Task: find a vector with length 1 pointing in the same direction as $\vec{v}$

$\vec{v} = \langle \sqrt{5}, 2, 4\rangle$



$$|\vec{v}|^2 = (\sqrt{5})^2 + 2^2 + 4^2$$

$$= 5 + 4 + 16 = 25$$

$$\overrightarrow{AB}$$

$$|\vec{v}| =$$