

1. Carothers 11.65, and the following follow up problem:

Show that if $\int_a^b |K(x, t)| dt \leq 1$ for all $x \in [a, b]$, then the Arzela-Ascoli theorem implies that given any function f , the sequence $(T^{(n)}(f))$ has a subsequence that converges in $C[a, b]$.

2. Suppose $f \in \mathcal{R}[a, b]$ and $\alpha \in \mathbb{R}$. Show that $\alpha f \in \mathcal{R}[a, b]$ and

$$\int_a^b \alpha f = \alpha \int_a^b f$$

3. Show that the uniform limit of Riemann integrable functions is Riemann integrable. Conclude that $\mathcal{R}[a, b]$ is a closed subset of $B[a, b]$.
4. Determine if $\chi_\Delta \in \mathcal{R}[0, 1]$, where Δ is the Cantor set.