Math F641: Homework 12 Due: November 24, 2021

**1.** Carothers 17.43
(The problem suggests you need to modify the proof of 17.20. Don't. Cite other results for a quick proof.)

**2.** Carothers 17.44

**3.** Carothers 17.45

**4.** Carothers 18.1

**5.** Carothers 18.3

**6.** Carothers 18.4

**7.** Carothers 18.6

**8.** Carothers 18.9

**9.** Carothers 18.11

**10.** Let $f \geq 0$ be Riemann integrable. In this exercise you will show that $f$ is measurable and that its Riemann integral $(R)\int_a^b f$ equals its Lebesgue integral $(L)\int_a^b f$. In your work, you are welcome to use the obvious fact that the Riemann integral and the Lebesgue integral agree for step functions.

1. Show that there exists a monotone increasing sequence of step functions $\phi_n$ and a monotone decreasing sequence of step functions $\psi_n$ such that $\phi_n \leq f \leq \psi_n$ for each $n$ and such that
$$(R)\int_a^b (\psi_n - \phi_n) \to 0.$$
2. Let $\Phi = \sup \phi_n$ and $\Psi = \inf \phi_n$. Show that $\Psi - \Phi = 0$ almost everywhere.
3. Conclude that $f$ is measurable.
4. Conclude that $(R)\int_a^b f = (L)\int_a^b f$.