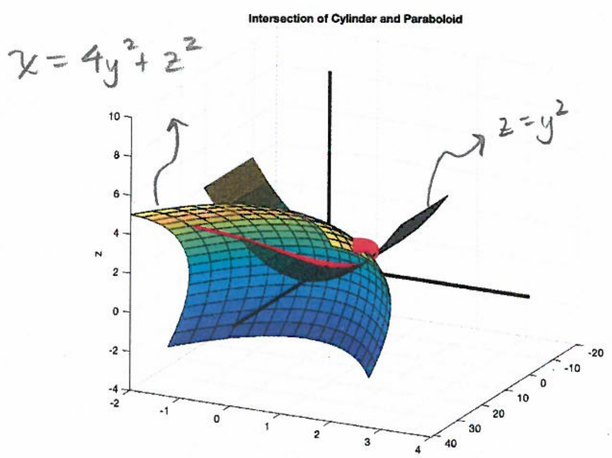
MATH 253
Quiz 3

Name: SOLUTIONS

January 29, 2021

Instructions: Five points total. Show all work for credit. **GS:** Scan two pages for your solutions.

1. (a) Find the vector-valued function $\mathbf{r}(\mathbf{t})$ that represents the curve of intersection between the paraboloid $x = 4y^2 + z^2$ and the cylinder $z = y^2$. See figure. Give your answer in vector form.



$\underbrace{z = y^2}$ $\quad\quad x = 4y^2 + z^2$

Slightly easier to start

Let $\boxed{y = t}$, $\boxed{z = t^2}$

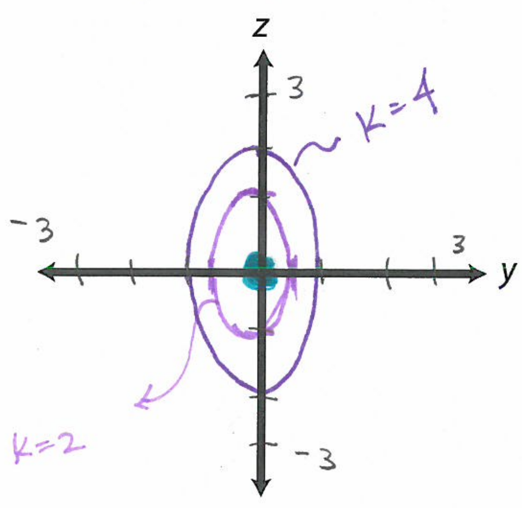
Then $x = 4y^2 + z^2$

$= 4t^2 + (t^2)^2$

$= \boxed{4t^2 + t^4 = x}$

Answer: $\mathbf{r}(t) = \left\langle t^4 + 4t^2,\ t,\ t^2 \right\rangle$, $t \in \mathbb{R}$.

(b) For the paraboloid $x = 4y^2 + z^2$, sketch the x-traces for the values of $k = 0, 1, 4$ on the axes below. Label the traces with their equations and include intercepts if relevant.



$k = 0, 1, 4$

origin $\boxed{K=0}$: $0 = 4y^2 + z^2 \leftrightarrow$ point $(0,0)$

$\boxed{K=1}$: $1 = 4y^2 + z^2$ ellipse

intercepts $(\pm\frac{1}{2}, 0)$, $(0, \pm 1)$

$\boxed{K=4}$: $4 = 4y^2 + z^2$ ellipse

intercepts $(\pm 1, 0)$ $(0, \pm 2)$

2. Consider the vector-valued function

$$\mathbf{r}(t) = \langle 0,\ t\,e^{3t},\ \cos^2(2t)\sin(2t)\rangle .$$

Compute the value of the definite integral

$$\int_0^{\frac{\pi}{2}} \mathbf{r}(t)dt$$

Let $x(t) = 0$, $y(t) = te^{3t}$, $z(t) = \cos^2(2t)\sin(2t)$ and

Compute $\int_0^{\pi/2} x(t)\,dt$, $\int_0^{\pi/2} y(t)\,dt$, $\int_0^{\pi/2} z(t)\,dt$

- $\int_0^{\pi/2} x(t)dt = 0$

- $\int_0^{\pi/2} te^{3t}\,dt \rightarrow$ Integration by parts. $u = t$ $dv = e^{3t}dt$ $du = dt$ $v = \frac{1}{3}e^{3t}$

$$= \int uv - vdu = \frac{1}{3}te^{3t} - \frac{1}{3}\int e^{3t}dt = \frac{1}{3}te^{3t} - \frac{1}{9}e^{3t}\Big|_0^{\pi/2}$$

$$= \left(\frac{\pi}{6}e^{\frac{3\pi}{2}} - \frac{1}{9}e^{3\pi/2}\right) - \left(0 - \frac{1}{9}\right) = \frac{\pi}{6}e^{3\pi/2} - \frac{1}{9}e^{\frac{3\pi}{2}} + \frac{1}{9}$$

- $\int_0^{\pi/2} \cos^2(2t)\sin(t)dt \rightarrow$ Substitution $u = \cos(2t)$ $du = -2\sin(2t)dt$

$\sin(2t)dt = \frac{-1}{2}du$

Limits: $t = 0 \Rightarrow u = 1$

$t = \pi/2 \Rightarrow u = \cos(\pi) = -1$

$$= \int_1^{-1} u^2\left[\frac{-1}{2}du\right] = \frac{-1}{2}\int_1^{-1} u^2du = -\frac{1}{6}u^3\Big|_1^{-1} = \frac{1}{6} - \left(-\frac{1}{6}\right) = \frac{1}{3}$$

Final Answer: $\boxed{\int_0^{\pi/2} \vec{r}(t)dt = \left\langle 0, \frac{\pi}{6}e^{3\pi/2} - \frac{1}{9}e^{3\pi/2} + \frac{1}{9}, \frac{1}{3}\right\rangle}$