MATH 253 Name: SOLUTIONS

Quiz 1 January 15, 2021

Instructions: Five points total. Show all work for credit. This quiz is closed-book, closed-notes, and no resources at all except for your brain and a pencil and piece of paper. Bald answers will receive no credit. Problem 2c is worth two points.

Gradescope instructions. Either print this quiz out and complete it, or give your answers on two blank pages. Scan two pages for your answers, as Gradescope will only accept solutions that are EXACTLY two pages in length. (This means not 1 page, not 3 pages, but only two pages.)

1. An particle is located at the point $P(-1, 5, -10)$, but is constrained so that it can only move in the straight-line direction toward the point $Q(3, 2, -4)$.

   Give, in coordinate form, the position vector $\overrightarrow{PQ}$ representing the direction in which the particle can move.

$\vec{PQ} = \vec{Q} - \vec{P} = \langle 3,2,-4\rangle - \langle -1,5,-10\rangle = \langle 3-(-1), 2-5, -4-(-10)\rangle = \langle 4,-3,6\rangle$

$\overrightarrow{PQ} =$ $\langle 4,-3,6\rangle$

2. Consider the two vectors $\mathbf{a} = \langle 2, 2\rangle$ and $\mathbf{b} = \langle -1-\sqrt{3}, 1-\sqrt{3}\rangle$ in $\mathbb{R}^2$.

   (a) Find the unit vector $\mathbf{u}$ pointing in the direction of $\mathbf{a}$.

$|\vec{a}| = \sqrt{2^2+2^2} = \sqrt{8} = 2\sqrt{2}$

$\vec{u} = \frac{1}{|\vec{a}|}\vec{a} = \frac{1}{2\sqrt{2}}\langle 2,2\rangle = \langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\rangle$

$\mathbf{u} =$ $\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\rangle$

   (b) Find the angle θ between the vectors $\mathbf{a}$ and $\mathbf{b}$. You final answer should be given as a 'well-known' angle.

$\theta = \arccos\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}\right)$

$|\vec{a}| = \boxed{2\sqrt{2}}$ from (a)

$|\vec{b}| = \sqrt{(-1-\sqrt{3})^2 + (1-\sqrt{3})^2}$

$= \sqrt{1+2\sqrt{3}+3 + 1-2\sqrt{3}+3}$

$= \sqrt{8} = \boxed{2\sqrt{2}}$

$\vec{a}\cdot\vec{b} = \langle 2,2\rangle \cdot \langle -1-\sqrt{3}, 1-\sqrt{3}\rangle$

$= -2-2\sqrt{3}+2-2\sqrt{3} = \boxed{-4\sqrt{3}}$

using the boxed values, we have

$\theta = \arccos\left(\frac{-4\sqrt{3}}{2\sqrt{2}(2\sqrt{2})}\right) = \arccos\left(\frac{-4\sqrt{3}}{8}\right)$

$= \arccos\left(\frac{-\sqrt{3}}{2}\right)$

$= \frac{5\pi}{6}$

[Ref angle: $\pi/6$, Quad II]

$\theta =$ $\frac{5\pi}{6}$

2pts

(c) Find the length of the component of **a** in the direction of **b** and the vector projection of **a** onto **b**. (If you failed to find θ in the last part, use $\theta = \frac{2\pi}{3}$ for this part.)

$$\text{Comp}_{\vec{b}}\,\vec{a} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$$

$$= \frac{-4\sqrt{3}}{2\sqrt{2}}$$

$$= -\frac{2\sqrt{3}}{\sqrt{2}}$$

$$= -\sqrt{6}$$

↑

will reverse direction of vector

$$\text{proj}_{\vec{b}}\,\vec{a} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\,\vec{b}$$

$$= \left(\text{comp}_{\vec{b}}\,\vec{a}\right)\frac{1}{|\vec{b}|}\,\vec{b}$$

$$= -\sqrt{6}\;\frac{1}{2\sqrt{2}}\;\langle -1-\sqrt{3},\ 1-\sqrt{3}\rangle$$

$$= \frac{-\sqrt{3}}{2}\;\langle -1-\sqrt{3},\ 1-\sqrt{3}\rangle$$

OR

$$\left\langle \frac{\sqrt{3}}{2}+\frac{3}{2},\ \frac{-\sqrt{3}}{2}+\frac{3}{2}\right\rangle$$

$\text{comp}_{\mathbf{b}}\,\mathbf{a} =$ $-\sqrt{6}$ $\qquad$ $\mathbf{proj}_{\mathbf{b}}\mathbf{a} =$ $\left\langle \frac{\sqrt{3}}{2}+\frac{3}{2},\ \frac{-\sqrt{3}}{2}+\frac{3}{2}\right\rangle$