Name:

1. The temperature on metal plate is given by

$$T(x, y) = 100e^{-(x^2+y^2)/100}$$

where T is measured in °C and x and y are measured in inches from the center of the plate.

1. Compute $\vec{\nabla} T(x, y)$.

$$\frac{\partial T}{\partial x} = 100e^{-(x^2+y^2)/100} \cdot \frac{(-2x)}{100}$$

$$= -2x\, e^{-(x^2+y^2)/100}$$

$$\frac{\partial T}{\partial y} = -2y\, e^{-(x^2+y^2)/100}$$

$$\boxed{\vec{\nabla} T = \left\langle \frac{\partial T}{\partial x}, \frac{\partial T}{\partial y} \right\rangle = -2e^{-(x^2+y^2)/100} \langle x, y \rangle \ {}^\circ\text{C/inch}}$$

2. At high noon a bug is standing at position $P(0, 1)$ and has velocity $\mathbf{v} = \langle -2, 1 \rangle$ inches/second.

   (a) What temperature does the bug see at high noon?

   $$T(0,1) = 100e^{-1/100} \ {}^\circ\text{C}$$

   (b) What is the rate of change in temperature that the bug sees at high noon?

   At $P(0,1)$, $\vec{\nabla} T = -2e^{-1/100} \langle 0, 1 \rangle$.

   Rate of change of temperature:

   $$\vec{\nabla} T \cdot \vec{v} = -2e^{-1/100} \langle 0, 1 \rangle \cdot \langle -2, 1 \rangle$$

   $$= -2e^{-1/100} \ {}^\circ\text{C/s}$$

2. Consider a position function $\mathbf{r}(t) = \left\langle \sin(2t, e^{-3t} - 1 \right\rangle$. For another function $T(x, y)$ you know that

$$T(0,0) = 7$$
$$T_x(0,0) = 3$$
$$T_y(0,0) = -2.$$

Compute

$$\frac{d}{dt} T(\mathbf{r}(t))$$

at $t = 0$.

$$\vec{r}(t) = \langle x(t), y(t) \rangle$$

$$\vec{r}\,'(t) = \left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle = \left\langle 2\cos(2t), -3e^{-3t} \right\rangle$$

$$\vec{r}\,'(0) = \langle 2, -3 \rangle$$

$$\frac{d}{dt} T(\vec{r}(t)) = \frac{\partial T}{\partial x}\frac{dx}{dt} + \frac{\partial T}{\partial y}\frac{dy}{dt}$$

$$= 3 \cdot 2 + (-2)(-3)$$

$$= 6 + 6$$

$$= \boxed{12}$$