Quiz 10
Math 253

Name: Key

11/15

1. A force field is given by $\mathbf{F}(x, y) = \langle x^2, xy \rangle$, which is *not* conservative. An object moves along a straight path from the point $(1, 1)$ to $(3, 2)$. Compute the work $\mathbf{F}$ did on the object.

$$\vec{r}(t) = \langle 1,1\rangle + t\langle 2,1\rangle = \langle 1+2t, 1+t\rangle \quad t \in [0,1]$$

$$\vec{r}\,'(t) = \langle 2,1\rangle$$

$$\int_C \vec{F}\cdot d\vec{r} = \int_0^1 \langle (1+2t)^2, (1+2t)(1+t)\rangle \cdot \langle 2,1\rangle \, dt$$

$$= \int_0^1 2(1+2t)^2 + (1+3t+2t^2)\, dt$$

$$= \frac{(1+2t)^3}{3} + t + \frac{3}{2}t^2 + \frac{2}{3}t^3 \Big|_0^1 = \left(9 + 1 + \frac{3}{2} + \frac{2}{3}\right) - \frac{1}{3}$$

$$= 10 + \frac{3}{2} + \frac{1}{3} = \frac{71}{6}$$

2. The field

$$\mathbf{F}(x, y) = \langle e^x \sin y + y^2 + 1, e^x \cos y + 2xy + y^2 \rangle$$

is conservative. Find *all* potential functions for it.

$$\frac{\partial f}{\partial x} = e^x \sin y + y^2 + 1$$

So $f(x,y) = e^x \sin y + xy^2 + x + C(y)$

So $$\frac{\partial f}{\partial y} = \cancel{e^x \cos y} + \cancel{2xy} + \frac{d}{dy}C(y) = \cancel{e^x \cos y} + \cancel{2xy} + y^2$$

$$\frac{d}{dy}C(y) = y^2$$

$$C(y) = \frac{y^3}{3} + D$$

$$f(x,y) = e^x \sin y + xy^2 + x + \frac{y^3}{3} + D$$