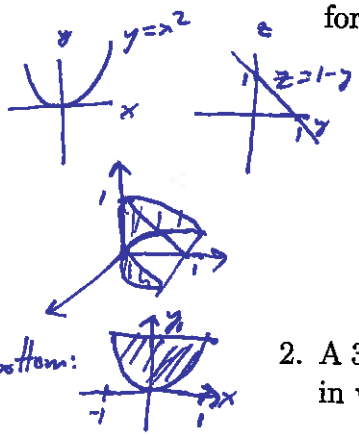
Quiz 9
Math 202 F01

Name: Solutions
4/1/2015

1. A 3-d region Q is bounded by the surfaces $y = x^2$, $z = 0$, and $z = 1 - y$. Give appropriate limits to express $\iiint_Q f(x, y, z)\, dV$ as an iterated integral. (You can't evaluate the integral, since you aren't given a formula for f.)



$$\int_{-1}^{1}\int_{x^2}^{1}\int_{0}^{1-y} f(x,y,z)\, dz\, dy\, dx$$

$$\left(\text{or } \int_{0}^{1}\int_{-\sqrt{y}}^{\sqrt{y}}\int_{0}^{1-y} f(x,y,z)\, dz\, dx\, dy\right)$$

2. A 3-d region Q is the quarter of the inside of the sphere $x^2+y^2+z^2 = 4$ in which $y \geq 0$ and $z \geq 0$.

   (a) Using spherical coordinates, give an iterated integral to compute

$$\iiint_Q z\, dV.$$

$$\int_0^{\pi}\int_0^{\pi/2}\int_0^{2} \underbrace{\rho\cos\phi}_{z}\ \underbrace{\rho^2\sin\phi\, d\rho\, d\phi\, d\theta}_{dV}$$

$$= \int_0^{\pi}\int_0^{\pi/2}\int_0^{2} \rho^3\cos\phi\sin\phi\, d\rho\, d\phi\, d\theta$$

   (b) Evaluate the integral in part (a).

$$\iiint_Q z\, dV = \int_0^{\pi}\int_0^{\pi/2} \frac{\rho^4}{4}\cos\phi\sin\phi\Big|_{\rho=0}^{2} d\phi\, d\theta$$

$$= \int_0^{\pi}\int_0^{\pi/2} 4\cos\phi\sin\phi\, d\phi\, d\theta = \int_0^{\pi} 2\sin^2\phi\Big|_0^{\pi/2} d\theta$$

$$= \int_0^{\pi} 2\, d\theta = 2\pi$$