MATH253X-UX1
Summer 2016

Midterm Exam 2

Name:______________________________

Instructions. (100 points) You have 90 minutes. Closed book, closed notes, no calculator. *Show all your work* in order to receive full credit.

(6$^{\text{pts}}$) **1.** Show that $\lim\limits_{(x,y)\to(-1,1)} \dfrac{xy+1}{2x^2-y^2-1}$ does not exist.

(8$^{\text{pts}}$) **2.** Use Lagrange multipliers to find the point(s) on the curve $x^2 - 2y^2 = 1$ closest from the point $P(0,2)$.

(6$^{\text{pts}}$) **3.** Find an equation of the tangent plane to the following surface at the point $(x_0, y_0, z_0) = (2, 1, -1)$:

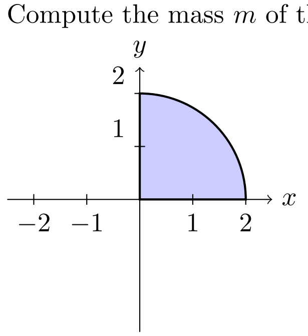
$$x \ln y - 3yz^2 + 1 = xz.$$

(12pts) **4.** For each of the iterated integrals below, sketch the region of integration then convert as indicated. DO NOT evaluate.

(a) (6 pts) Rewrite $\int_{-2}^{0} \int_{0}^{x^2} 3xy \, dy \, dx$ in the order $dx \, dy$.

(b) (6 pts) Rewrite $\int_{-\frac{\pi}{2}}^{\frac{\pi}{4}} \int_{0}^{1} r^2 \, dr \, d\theta$ in rectangular coordinates.

(12pts) **5.** Compute the mass m of the planar lamina with density $\rho(x, y) = y^2$ shown below.



(16pts) **6.** Consider the function:

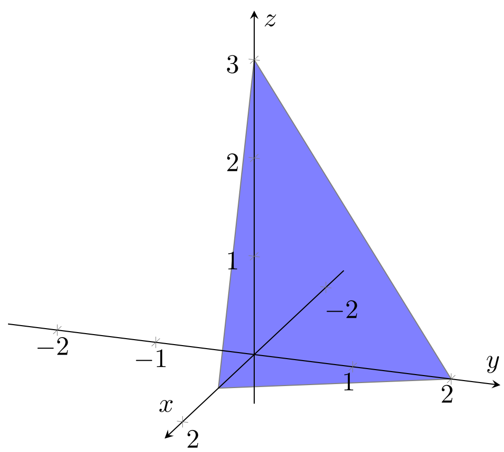
$$f(x, y) = x^3 - 12xy + 8y^3.$$

(a) (8 pts) Find and classify all critical points of $f(x, y)$.

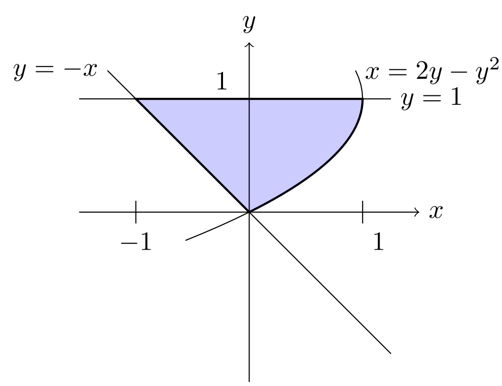
(b) (8 pts) Find the absolute minimum and maximum values of $f(x, y)$ in the rectangular region R defined by $0 \leq x \leq \frac{1}{2}$ and $0 \leq y \leq 1$.

(22pts) **7.** Evaluate the following.

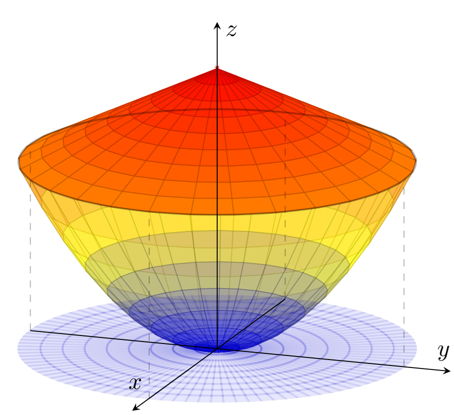
(a) (8 pts) the volume below the plane $6x + 3y + 2z = 6$ in the first octant:



(b) (6 pts) the surface area of the cone $z = \sqrt{x^2 + y^2}$ above the region R bounded by the graphs of $y = -x$, $x = 2y - y^2$, $y = 0$ and $y = 1$ as sketched below:



(c) (8 pts) the volume of the solid bounded by the paraboloid $z = x^2 + y^2$ and the inverted cone $z = 6 - \sqrt{x^2 + y^2}$ using polar coordinates.



(18pts) **8.** The bee population in a boxed beehive is given at each point (x, y, z) by

$$f(x, y, z) = x^2 + y^2 + xyz.$$

(a) (6 pts) At the point $(3, 1, 2)$, what is the unit direction of greatest decrease in population?

(b) (6 pts) Find the directional derivative of f at $(3, 1, 2)$ in the direction of $\mathbf{v} = \langle 1, 2, 2\rangle$?

(c) (6 pts) Use the chain rule (no direct substitution) to find $\frac{df}{dt}$ in terms of t if $x(t) = 4-t^2$, $y(t) = 3t-2$ and $z(t) = 3t^3 - 1$.