

# Math F253

# Midterm 1

# Fall 2021

Name: \_\_\_\_\_

Section: F02 (Maxwell)

Student Id: \_\_\_\_\_

Calculator Model: \_\_\_\_\_

## Rules:

You have 60 minutes to complete the exam.

Partial credit will be awarded, but you must show your work.

A scientific or graphing calculator (without symbolic manipulation) is allowed.

A one page sheet of paper (8 1/2 in. x 11 in.) with handwritten notes on one side is allowed.

No other aids are permitted.

Place a box around your **FINAL ANSWER** to each question where appropriate.

If you need extra space, you can use the back sides of the pages. Please make it obvious when you have done so.

Turn off anything that might go beep during the exam.

Good luck!

| Problem      | Possible | Score |
|--------------|----------|-------|
| 1            | 12       |       |
| 2            | 12       |       |
| 3            | 12       |       |
| 4            | 5        |       |
| 5            | 12       |       |
| 6            | 10       |       |
| 7            | 12       |       |
| Extra Credit | 3        |       |
| Total        | 75       |       |

**1. (12 points)**

Consider the vectors  $\mathbf{v} = \langle 1, -2, -4 \rangle$  and  $\mathbf{w} = \langle 2, 3, -2 \rangle$ .

a. Is the angle between the two vectors acute, obtuse, or right? Justify your answer.

b. Find a unit vector pointing in the same direction as  $\mathbf{w}$ .

c. Compute the area of the parallelogram with sides  $\mathbf{v}$  and  $\mathbf{w}$ .

**2. (12 points)**

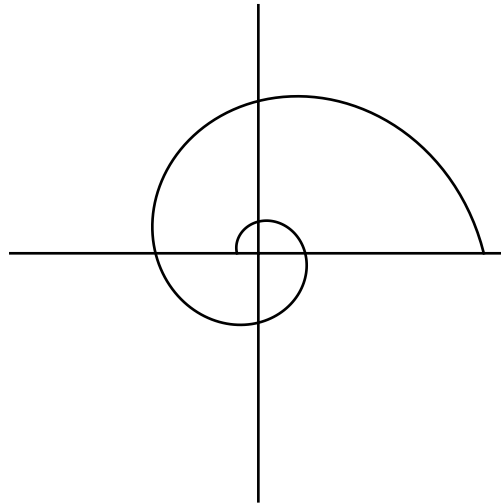
A rocket experiences two forces in units of Newtons:

- gravitational,  $\mathbf{F}_g = -2000\mathbf{k}$
- thrust,  $\mathbf{F}_T = 1000\mathbf{i} + 4000\mathbf{k}$

- a. Compute the total force  $\mathbf{F}$  the rocket experiences.
- b. Make a diagram with arrows that illustrates both the two forces  $\mathbf{F}_g$  and  $\mathbf{F}_T$  as well as the total force  $\mathbf{F}$ . Your diagram should be reasonably accurate.
- c. In a short time interval the rocket experiences the constant total force  $\mathbf{F}$  you computed in part a as it moves from  $P(10, 5, 20)$  to  $Q(13, 5, 26)$ ; all units are in meters. Compute the work done on the rocket by gravity and thrust combined.

**3. (12 points)**

The figure below contains a sketch of the curve parameterized by  $\mathbf{r}(t) = \langle e^{-t/4} \cos(t), e^{-t/4} \sin(t) \rangle$  for  $0 \leq t \leq 3\pi$ .



- Label the point  $P = \mathbf{r}(\pi/2)$  in the diagram.
- Sketch the tangent **line** at  $t = \pi/2$ .
- Find a formula for the tangent line  $\vec{\ell}(s)$  at  $t = \pi/2$ .

Continued....

Problem 3 continued....

- d. Find a parameter value  $t$  such that the tangent line to the curve  $\mathbf{r}(t)$  is horizontal.

4. (5 points)

You are using a wrench to tighten a lug nut on a car tire. So you are turning the wrench clockwise. In what direction is the torque vector you are generating pointing? Explain.



**6. (12 points)**

Consider the curve

$$\mathbf{r}(t) = \langle \sqrt{2} \sin(4t), \cos(4t), \cos(4t) \rangle.$$

- a. Compute  $\mathbf{r}'(t)$
  
  
  
  
  
  
  
  
  
  
- b. Find the speed of the curve as a function of  $t$ .
  
  
  
  
  
  
  
  
  
  
- c. Compute the length of the curve from  $t = 0$  to  $t = \pi/4$ .

**7. (10 points)**

A particle moves with acceleration  $\mathbf{a}(t) = \left\langle 1, \frac{t}{1+t^2}, 2t \right\rangle$  in units of  $\text{m/s}^2$ . At  $t = 0$  the particle has velocity  $\mathbf{v}(0) = \langle 1, 0, -2 \rangle \text{ m/s}$ .

a. Determine  $\mathbf{v}(t)$ .

b. Determine all times the velocity is horizontal.

**8. (Extra Credit: 3 points)**

In problem 3a the curve has a familiar shape and is contained in a plane. Find the shape and the plane.