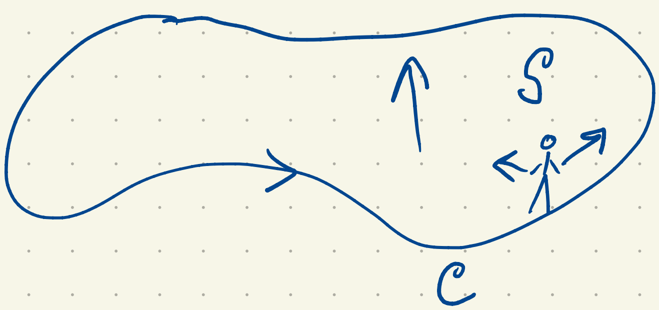
Last class:

Stoke's Theorem



$$\int_C \vec{Z}\cdot d\vec{r} = \iint_S (\vec{\nabla}\times\vec{Z})\cdot\vec{n}\, dS$$

$$\vec{\nabla}\times\vec{Z}\cdot d\vec{S}$$

weird, means same thing.

$$F(b)-F(a) = \int_a^b F'(x)\,dx$$

$= 8 - 2 + 12 = 18$

net: 17

$x^2+y^2+z^2=4 \qquad x^2+y^2 \le 1 \Rightarrow z^2 \geq 3$

$\vec{F} = xz\hat{i} + yz\hat{j} + xy\hat{k}$

$\iint \vec{\nabla}\times\vec{F}\cdot\vec{n}\,dS \qquad \vec{r} = \langle u, v, \sqrt{4-u^2-v^2}$

$z = \sqrt{4-x^2-y^2}$

$\vec{r}_u \times \vec{r}_v = \left\langle \frac{-u}{\sqrt{4-u^2-v^2}}, \frac{-v}{\sqrt{4-u^2-v^2}}, 1 \right\rangle$

$\partial_x \quad \partial_y \quad \partial_z$

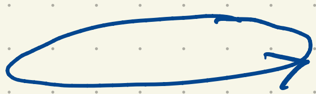
$xz \quad yz \quad xy$

$\langle (x-y), (x-y), 0 \rangle$

$\vec{\nabla}\times F = \langle u-v, u-v, 0 \rangle$

$$\vec{\nabla}\times\vec{F}\cdot \vec{r}_u\times\vec{r}_v = \frac{-u^2+uv-uv+v^2}{\sqrt{4-u^2-v^2}}$$

$$\int_0^1\int_0^{2\pi} \frac{r^2}{\sqrt{4-r^2}}\left[-\cos^2\theta+\sin^2\theta\right] d\theta\, dr = 0.$$



$$\vec{r}(s) = \langle \cos(s), \sin(s), \sqrt{3} \rangle$$

$$\vec{r}\,'(s) = \langle -\sin(s), \cos(s), 0 \rangle$$

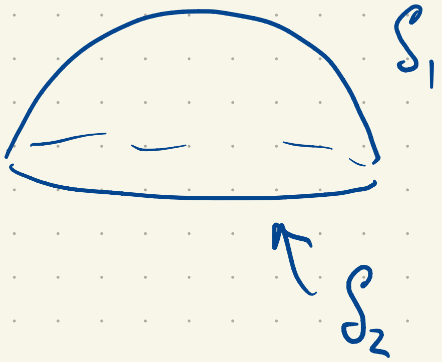
$$\vec{F}(\vec{r}(s)) = \langle \sqrt{3}\cos(s), \sqrt{3}\sin(s), \cos(s)\sin(s) \rangle$$

xz yz xy

$$\vec{F}\cdot\vec{r}\,'(s) = \sqrt{3}\left(-\sin(s)\cos(s)+\cos(s)\sin(s)\right)$$

$$=0 \quad (!)$$

$$\int_C \vec{F}\cdot d\vec{r} = 0.$$

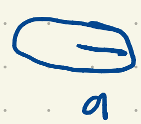


$$\iint_{S_1} \vec{\nabla}\times\vec{F}\cdot\vec{n}\, dS = \iint_{S_2} \vec{\nabla}\times\vec{F}\cdot n$$

$$\hat{k}$$

$$\vec{\nabla}\times\vec{F} = \langle x-y,\, x-y,\, 0\rangle$$

$$\vec{\nabla}\times\vec{F}\cdot\hat{k} = 0$$



$$\left(\vec{\nabla}\times\vec{V}\cdot\vec{n}\right)\cdot\pi a^2 \approx \int_C \vec{V}\cdot d\vec{r}$$

$$\underbrace{\frac{1}{2\pi a}\int_C (\vec{V}\cdot T)\, ds}_{\text{average tangential velocity}}$$

time needed to go around?

$$\frac{1}{(2\pi a)^2}\int_C (\vec{V}\cdot T)\,ds = \frac{1}{(2\pi a)^2}\iint_{C} (\nabla\times V)\cdot\vec{n}$$

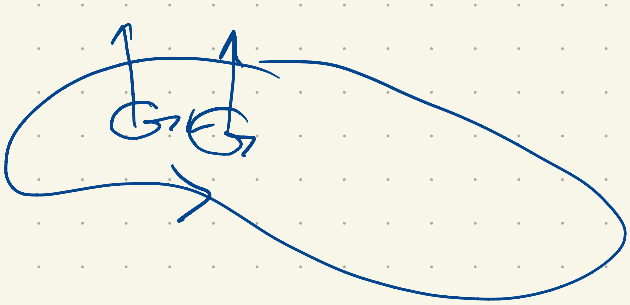
$$= \frac{1}{4\pi}\,\frac{1}{\pi a^2}\iint_{D}$$

$$\rightarrow \frac{1}{4\pi}(\nabla\times V)\cdot\vec{n}$$

$\frac{1}{4\pi}\nabla\times V$ is the ang. velocity (rotations per second) the fluid is rotating at a location

$\frac{\text{rot}}{s}\cdot\frac{2\pi\text{ rad}}{1\text{ rot}}$

$$\boxed{\frac{1}{2}\nabla\times V \text{ is ang velocity in radians / time}}$$

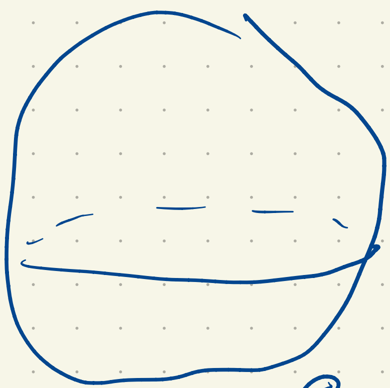


interior circulations cancel.

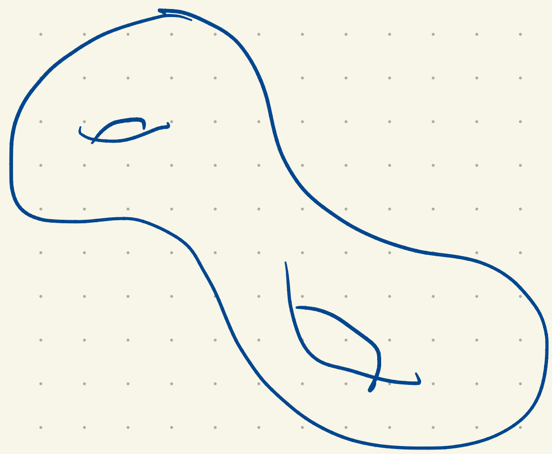
boundary remains.

Another cousin of the FTC

Divergence Theorem

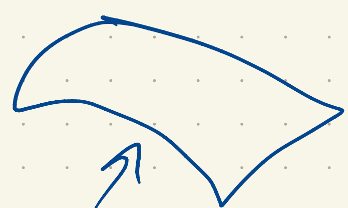


S



S

S has to enclose a region $\mathcal{E}$.

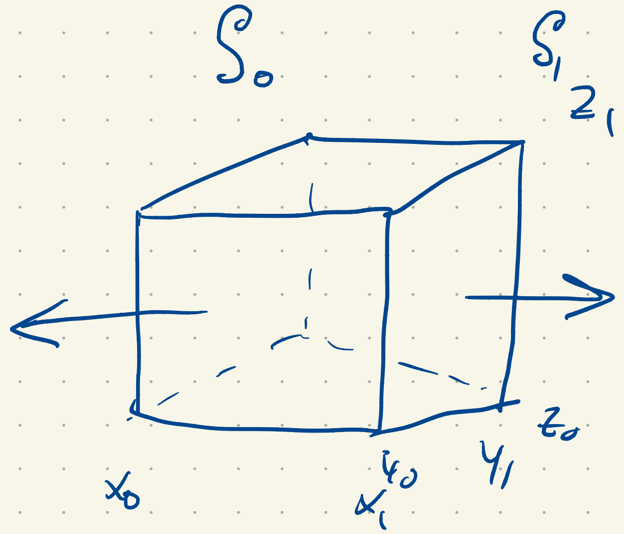


not ok.

Surface is always orientable; we pick $\vec{n}$ pointing to exterior.

$$\boxed{\iint_S \vec{X}\cdot\vec{n}\,dS = \iiint_{\mathcal{E}} \operatorname{div}\vec{X}\,dV}$$

e.g.



$$\vec{X} = P\hat{i} + Q\hat{j} + R\hat{k}$$

$$\iiint_{\mathcal{E}} \frac{\partial P}{\partial x} = \int_{z_0}^{z_1}\int_{y_0}^{y_1}\int_{x_0}^{x_1} \frac{\partial P}{\partial x}\, dx\, dy\, dz$$

$$= \int_{z_0}^{z_1}\int_{y_0}^{y_1} \left(P(x_1, y, z) - P(x_0, y, z)\right)$$

On S_1, $\vec{X}\cdot\vec{n} = P$

On S_2 $\vec{X}\cdot\vec{n} = -P$

$$= \iint_{S_1} \vec{X}\cdot\vec{n}\, dS + \iint_{S_0} \vec{X}\cdot\vec{n}\, dS$$

Remains 4 sides come from

$$\iiint \frac{\partial Q}{\partial y} \text{ and } \iiint \frac{\partial R}{\partial z}$$