$$X = P\hat{i} + Q\hat{j} + R\hat{k}$$

$$X \cdot \vec{r}_u \times \vec{r}_v = \left(-Pf_u - Qf_v + R\right)$$

$$f_u = -2u \qquad f_v = -2v$$

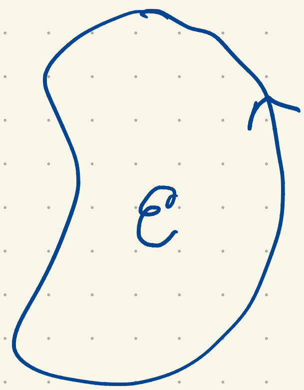
$$\rho \iint_D -u\cdot(-2u) - v(-2v) + 2(1-u^2-v^2)\ dA(u,v)$$

$$\rho \iint_D \left[2(u^2+v^2) + 2 - 2(u^2+v^2)\right] dA(u,v)$$

$$= \rho \iint_D 2\ dA = \underbrace{2\pi\rho}$$

$\hookrightarrow$ mass flux

Recall Green's Thm



$$\iint_{\mathcal{E}} -\frac{\partial P}{\partial y} + \frac{\partial Q}{\partial x}\, dA(x,y) = \int_{\mathcal{C}} P\,dx + Q\,dy = \int_{\mathcal{C}} \vec{F}\cdot d\vec{r}$$

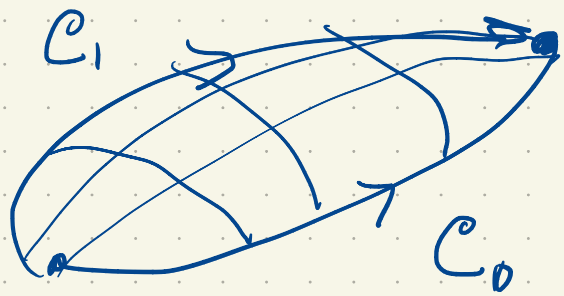
$$F = \langle P, Q, R \rangle$$

$$\vec{\nabla}\times\vec{F} = 0\,\hat{\imath} + 0\,\hat{\jmath} + \left(-\frac{\partial P}{\partial y} + \frac{\partial Q}{\partial x}\right)\hat{k}$$

$$\int_{\mathcal{C}} \vec{F}\cdot dr = \iint_{S} \left(\vec{\nabla}\times\vec{F}\cdot \overset{s}{\hat{n}}\right) dS$$

This holds for any orientable simply connected surface S so long as when looking at the boundary from "above", ($\overset{s}{\hat{n}}$ pointing at you), $\mathcal{C}$ goes c.c.w.

This is another form of the FTC



derivatives of F

$$\int_{C_1} F\cdot dr = \int_{C_1} F\cdot dr + \iint_S$$

$$\int_C \vec{F}\cdot dr = \iint_S \left(\vec{\nabla}\times\vec{F}\cdot\vec{n}\right) dS$$

~~$= 8 - 2 + 12 = 18$~~

~~ref: 17~~

$x^2+y^2+z^2=4 \qquad x^2+y^2 \le 1 \Rightarrow z^2 \ge 3$

$\vec{F} = xz\hat{i} + yz\hat{j} + xy\hat{k}$

$\iint \vec{\nabla} \times \vec{F} \cdot \vec{n}\, dS \qquad \vec{r} = \langle u, v, \sqrt{4-u^2-v^2}$

$z = \sqrt{4-x^2-y^2}$

$\vec{r}_u \times \vec{r}_v = \left\langle \frac{-u}{\sqrt{4-u^2-v^2}}, \frac{-v}{\sqrt{4-u^2-v^2}}, 1 \right\rangle$

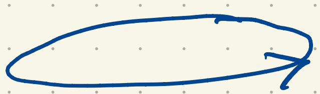
$$\begin{matrix} \partial_x & \partial_y & \partial_z \\ xz & yz & xy \end{matrix}$$

$\langle (x-y), (x-y), 0 \rangle$

$\vec{\nabla} \times F = \langle u-v, u-v, 0 \rangle$

$$\vec{\nabla}\times\vec{F}\cdot\vec{r}_u\times\vec{r}_v = \frac{-u^2+uv-uv+v^2}{\sqrt{4-u^2-v^2}}$$

$$\int_0^1\int_0^{2\pi}\frac{r^2}{\sqrt{4-r^2}}\left[-\cos^2\theta+\sin^2\theta\right]d\theta\,dr = 0.$$



$$\vec{r}(s) = \langle \cos(s), \sin(s), \sqrt{3}\rangle$$

$$\vec{r}\,'(s) = \langle -\sin(s), \cos(s), 0\rangle$$

$$\vec{F}(\vec{r}(s)) = \langle \sqrt{3}\cos(s), \sqrt{3}\sin(s), \cos(s)\sin(s)\rangle$$

xz yz xy

$$\vec{F}\cdot\vec{r}\,'(s) = \sqrt{3}\left(-\sin(s)\cos(s)+\cos(s)\sin(s)\right)$$

$$= 0 \quad (!)$$

$$\int_C \vec{F}\cdot d\vec{r} = 0.$$