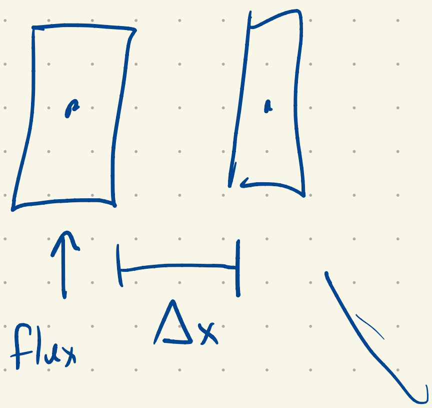
16.5

Divergence

$$\vec{V} = P\hat{i} + Q\hat{j} + R\hat{k}$$

↑

imagine as a velocity of a fluid

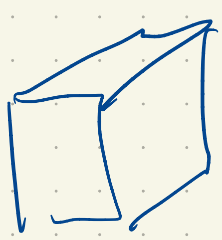


is ≈

$$P(x,y,z)\cdot\Delta y\Delta z - P(x+\Delta x,y,z)\cdot\Delta y\Delta z$$

Net flux in: $-\dfrac{\partial P}{\partial x}\Delta x\Delta y\Delta z$

Ditto in other directions



$$-\left[\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}\right]\Delta x\Delta y\Delta z$$

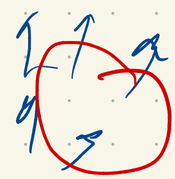
↑

divergence of $\vec{V}$, $\operatorname{div}\vec{V}$ $\vec{\nabla}\cdot\vec{V}$

You should think of this as the amount of fluid per volume leaving a region, per unit time.

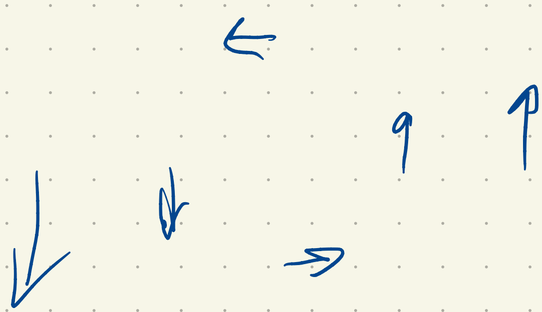
e.g. $\vec{V} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\vec{\nabla}\cdot\vec{V} = 3$$



No matter where you look, more leaves than is generated.

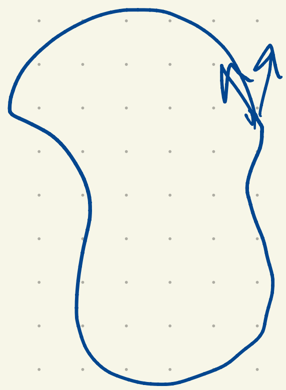
$$\vec{V} = -y\hat{i} + x\hat{j}$$



$$\vec{\nabla}\cdot\vec{V} = 0$$

In 2-d: $\vec{\nabla}\cdot\vec{V} = \dfrac{\partial P}{\partial x} + \dfrac{\partial Q}{\partial y}$

Green's thm in terms of divergence:



$$\int_C \vec{F} \cdot \vec{n} \, ds = \int_C$$

$$\vec{F} = P\hat{i} + Q\hat{j}$$

$$T = \frac{x'}{|\vec{r}'|}\hat{i} + \frac{y'}{|\vec{r}'|}\hat{j}$$

$$n = \frac{y'}{|\vec{r}'|}\hat{i} - \frac{x'}{|\vec{r}'|}\hat{j}$$ outward pointing

$$\int \vec{F} \cdot \vec{n} \, ds = \int \left(\frac{P y'}{|\vec{r}'|} - \frac{Q}{|\vec{r}'|} x' \right) |\vec{r}'| \, dt$$

$$= \int -Q x' + P y' \, dt$$

$$= \int -Q \, dx + P \, dy$$

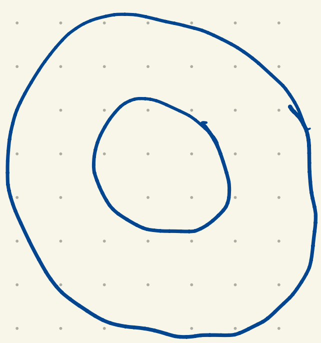
$$= \iint + \frac{\partial Q}{\partial y} + \frac{\partial P}{\partial x}\, dA$$

$$= \iint_{\mathcal{R}} (\operatorname{div} F)\, dA$$

$$\boxed{\iint_{\mathcal{R}} \operatorname{div} F\, dA = \int_C \vec{F} \cdot \vec{n}\, ds}$$

2-d Divergence thm.

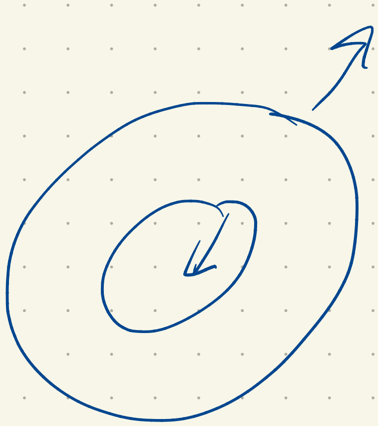
e.g. $F = \frac{x}{|\vec{x}|^2}\hat{i} + \frac{y}{|\vec{x}|^2}\hat{j}$



$$\vec{\nabla} \cdot \vec{F} = \frac{1}{(x^2+y^2)} - \frac{x}{(x^2+y^2)^2} \cdot 2x$$

$$+ \frac{1}{(x^2+y^2)} - \frac{2y}{(x^2+y^2)^2}$$

$$= \frac{-x^2+y^2 \quad + x^2 - y^2}{(\quad)} = 0$$



Outer flux through C_0

= Outer flux thru C_1