$\vec{r}(t)$

$$\int_{t_0}^{t_1} \left(M(\vec{r}(t))\, x' + N(\vec{r}(t))\, y' + P(\vec{r}(t))\, z' \right) dt$$

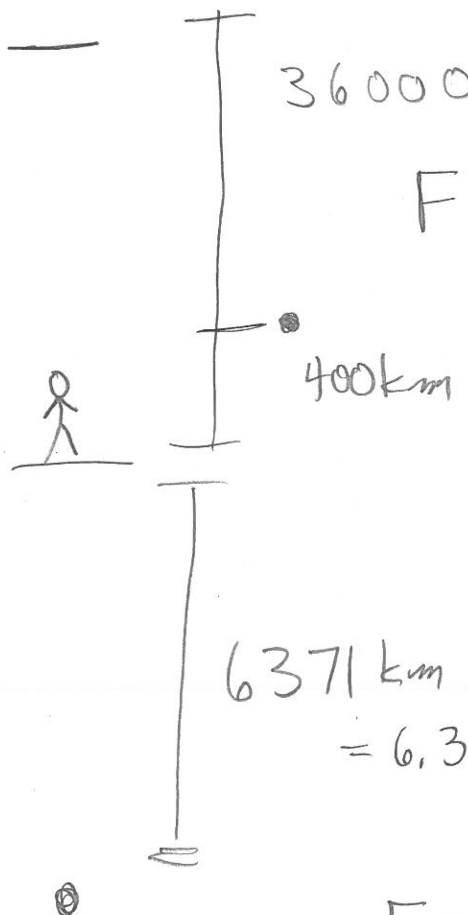
$$\int_{t_0}^{t_1} \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\, dt$$

$$\langle M, N, P \rangle \cdot \langle x', y', z' \rangle$$

$$\int_C \vec{F} \cdot d\vec{r} = \int_C \left(M\,dx + N\,dy + P\,dz \right)$$

$$\int_0^1 \underset{x'}{4t, 1} + \underset{y'}{5t, 4} + \underset{z'}{(z+t) \cdot 5} \, dt$$

How much work to put David in orbit?



$= 6.371 \times 10^6$ m

5.97

$M = \cancel{6} \times 10^{24}$ kg

$$F = -\frac{GMm}{z^2}\hat{k}$$

$$G = 6.67 \times 10^{-11} \frac{m^3\ s}{kg\ s}$$

$$GM = 3.98 \times 10^{14}$$

$$GM \cdot m \left(\frac{1}{r_1} - \frac{1}{r_0} \right) =$$

$$-3.69 \times 10^{8}\ \text{J}$$

$$\int_{t_0}^{t_1} \frac{-GMm}{|x|^3}\, x \cdot r'\, dt$$

$$r(t) = \langle \cancel{6371}\, t\, i \rangle \qquad 6371 \le t \le 6771 \qquad \cancel{0 \le t \le 400}$$

$$\int_{\cancel{0}\,6371}^{6771\ \cancel{400}} \frac{-GMm}{|r(t)|^3}\, r \cdot \vec{r}'\, dt$$

$$-GMm \int \frac{t}{t^3}\, dt = GMm \left(\frac{1}{t} \right) \Bigg|_{6371}^{6771}$$

$$= GMm \left(\frac{1}{6771} - \frac{1}{6371} \right)$$

$$\frac{GMm}{|\vec{x}|} \qquad \frac{GMm}{\sqrt{x^2+y^2+z^2}}$$

$$\nabla f = \frac{-GMm}{|\vec{x}|^3}\vec{x}$$

↑

force due to gravity,

$$\int_C \vec{F}\cdot d\vec{r} = f(B) - f(A)$$

Is this an accident?

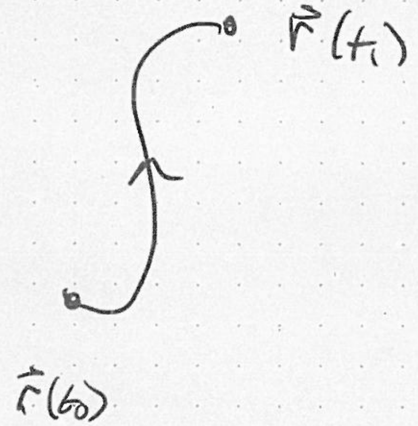
What is the integral of a gradient?

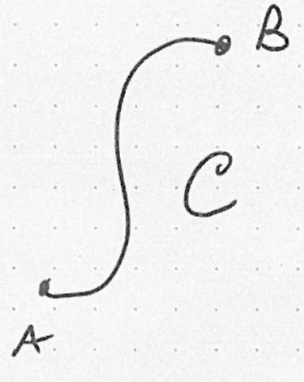
$$\int_C \nabla f \cdot d\vec{r} = \int_{t_0}^{t_1} \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right\rangle \cdot \langle x', y', z' \rangle \, dt$$

$$= \int_{t_0}^{t} \frac{\partial f}{\partial x} x' + \frac{\partial f}{\partial y} y' + \frac{\partial f}{\partial z} z' \, dt$$

$$= \int_{t_0}^{t_1} \frac{d}{dt} f(\vec{r}(t)) \, dt$$

$$= f(\vec{r}(t_1)) - f(\vec{r}(t_0))$$





$$\int_C \nabla f \cdot d\vec{r} = f(B) - f(A)$$

Conversely if $\int_C \vec{F} \cdot d\vec{r} = 0$ around

every loop C then $\int_{C_1} \vec{F} \cdot d\vec{r} = \int_{C_2} \vec{F} \cdot d\vec{r}$

is path independent

If $F = \nabla f$ then

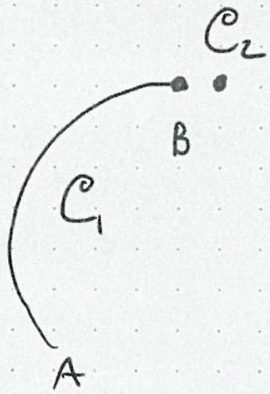
1) $\int_C \vec{F} \cdot d\vec{r}$ depends only on endpoints of C

2) $\int_C \vec{F} \cdot d\vec{r} = 0$ $\forall$ loops C.

[1) $\Leftrightarrow$ 2)]

Is the converse true?

$$f(x,y) = \int_C \vec{F}\cdot d\vec{r}$$ any path from A to (x,y)



$$f(x+h,y) = \int_{C_2} \vec{F}\cdot d\vec{r}$$

$$= \int_{C_2} P\,dx + Q\,dy$$

$$\vec{r}(t) = \langle t, y \rangle \qquad x \le t \le x+h$$

$$\int_{C_2} \vec{F}\cdot d\vec{r} = \int_{x_0}^{\cancel{x+h}\; x} P(t, y_0)\,dt$$

$$\left.\frac{\partial}{\partial h}\right|_{h=0} = P(x_0, y_0)$$

Similarly $\quad \dfrac{\partial f}{\partial y} = Q.$ $\quad$ I.e. $P\,dx + Q\,dy$

$$\frac{\partial f}{\partial x}\,dx + \frac{\partial f}{\partial y}\,dy$$

$$\boxed{\vec{F} = \vec{\nabla} f.}$$

Path ind $\Rightarrow$ can find a potential.

How can you tell?

$\vec{F} = \langle P, Q \rangle$ $\vec{\imath}$ P, Q cts and have cts derivatives

then $P = \frac{\partial f}{\partial x}$ $Q = \frac{\partial f}{\partial y}$

$\Rightarrow$ $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ necessary.

Is it sufficient?