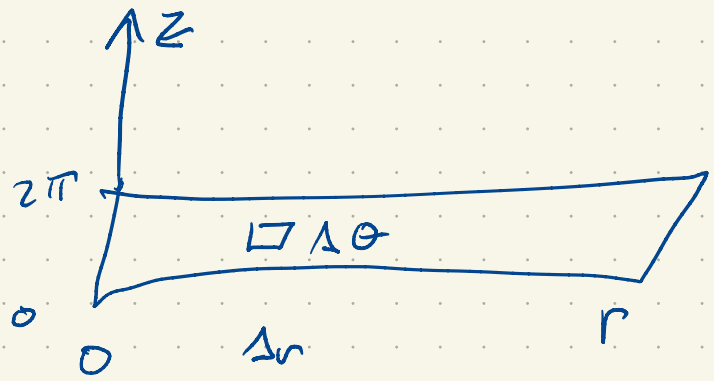
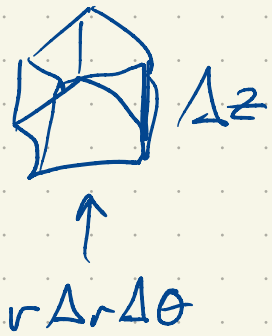
Cylindrical coord





$x = r\cos\theta$ $\quad r = \sqrt{x^2+y^2}$

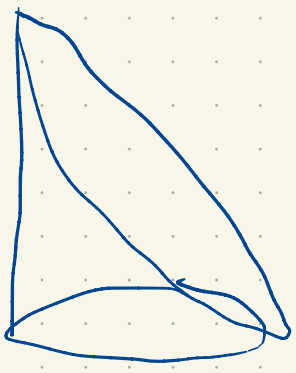
$y = r\sin\theta$

$z = z$

Integrate $\sqrt{x^2+y^2}$ over region bounded by $x^2+y^2=16$

$z = 0$

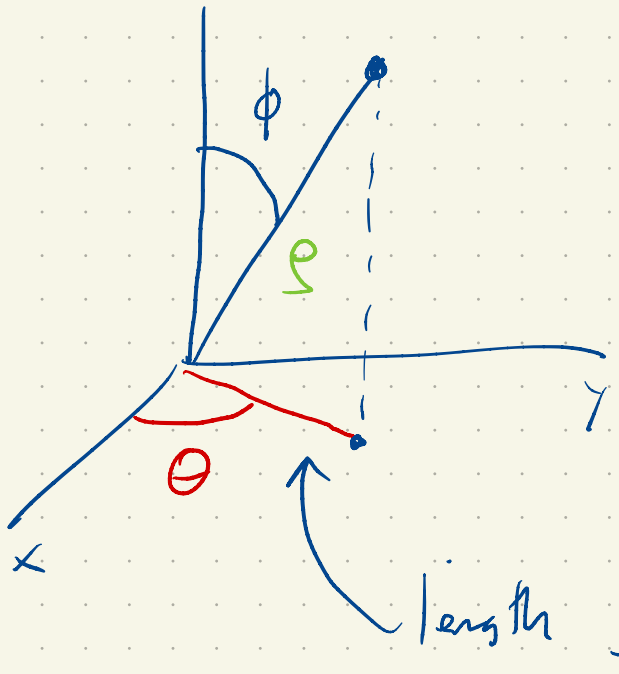
$z = 4-y$



$$\int_0^{2\pi}\int_0^{4}\int_0^{4-r\sin\theta} r \; dz\, r\,dr\,d\theta$$

$$2\pi \int_0^4 r\,(4 - r\sin\theta)\, r\, dr = \frac{512\pi}{3}$$

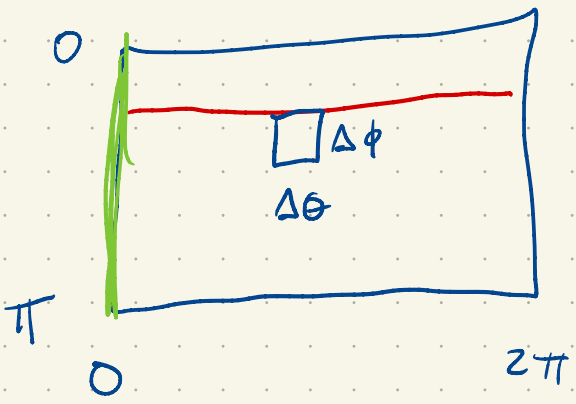
Spherical

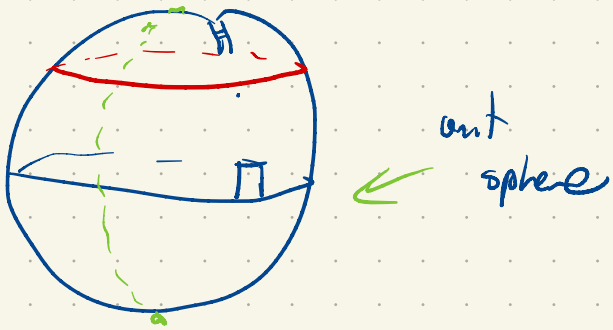


$z = \rho\cos\phi$

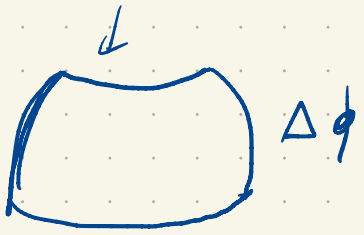
$0 \le \phi \le \pi$

$x = \rho\sin\phi\cos\theta \quad y = \rho\sin\phi\sin\theta$

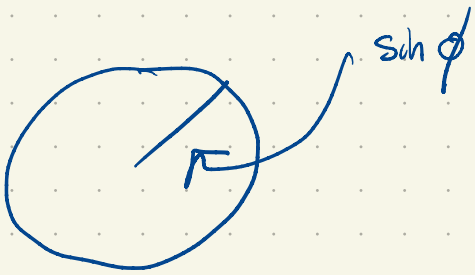




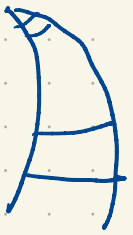
$\sin(\phi+\Delta\phi)\Delta\theta \approx \sin(\phi)\Delta\theta$



$\sin\phi\,\Delta\theta$



Area: $\sin\phi\,\Delta\phi\Delta\phi$



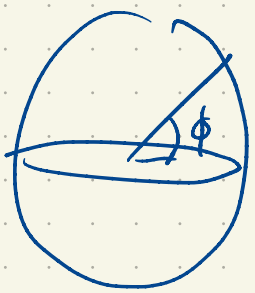
For a spree of radius R:

$R\sin\phi\Delta\theta,\ R\Delta\phi$

$R^2\sin\phi\Delta\theta\Delta\phi$

$dV = \rho^2\sin\phi\ d\phi\,d\theta\,d\rho$

Alt:



$$z = \rho \sin\phi$$

$$x = \rho \cos\phi \cos\theta$$

$$y = \rho \cos\phi \sin\theta$$

$$dV = \rho^2 \cos\phi \, d\rho \, d\theta \, d\phi$$

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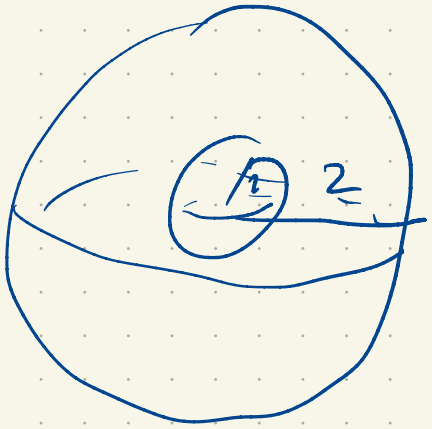
e.g.



$$\iiint_E z^2 \, dV$$

$$\int_0^{2\pi} \int_0^{\pi} \int_1^2 \rho^2 \cos^2\phi \; \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

$$\int_0^{2\pi} \int_0^{\pi} \frac{\rho^5}{5} \Big|_1^2 \cos^2\phi \sin\phi \, d\phi \, d\theta$$

$$2\pi \left(\frac{2^5 - 1}{5} \right) \int_0^{\pi} \cos^2\phi \sin\phi \, d\phi \qquad u = \cos\phi$$

$$\frac{124}{15}\pi$$