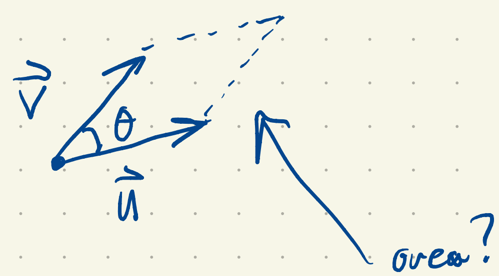
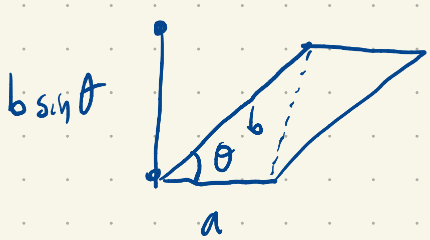
Section 15.6

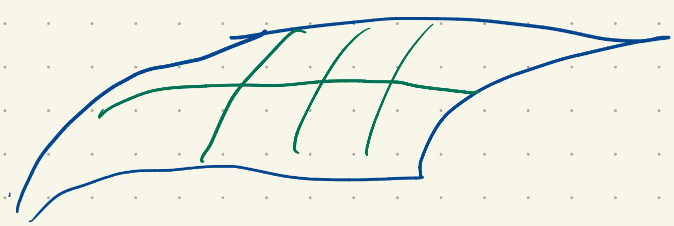
Surface Area

Ok, fine. Integrals are sometimes about areas and volumes.

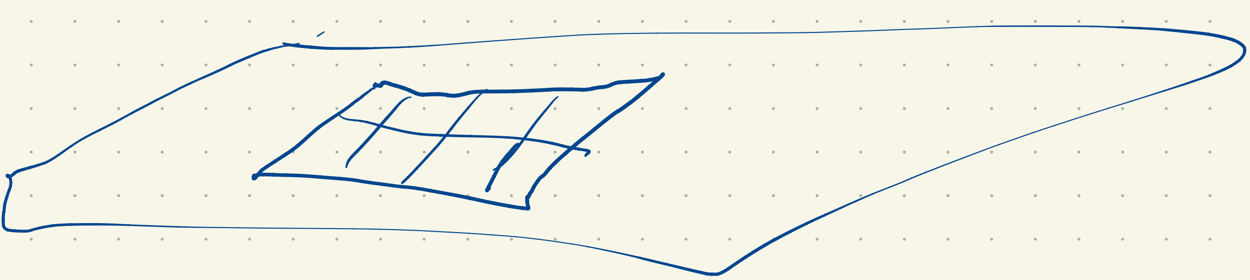


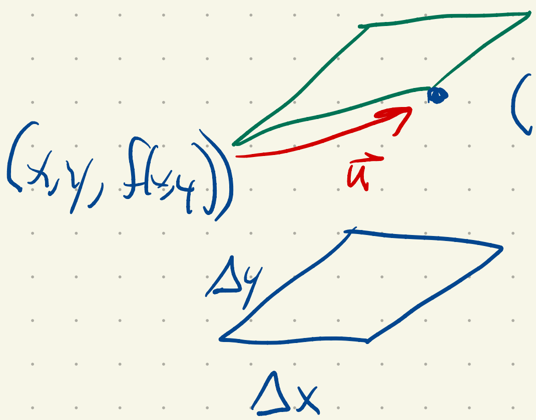
$$\left|\vec{u} \times \vec{v}\right| = |\vec{u}||\vec{v}| \sin\theta$$





$z = f(x)$





$$(x+\Delta x, y, f(x,y) + f_x \Delta x + \overset{o(\Delta x)}{\varepsilon}$$

$$= (x,y,f(x,y)) + (\Delta x, 0, f_x \Delta x)$$

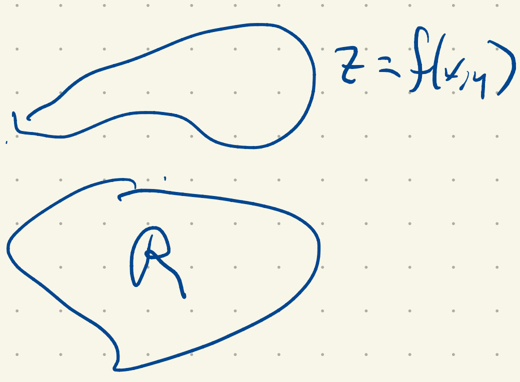
$$\vec u = (\Delta x, 0\ f_x \Delta x)$$

$$\vec v = (0, \Delta y, f_y \Delta x)$$

$$\vec u \times \vec v = (-f_x \Delta x \Delta y, -f_y \Delta x \Delta y, \Delta x \Delta y)$$

$$|\vec u \times \vec v| = (1 + f_x^2 + f_y^2)^{1/2} \Delta x \Delta y$$

$$\sum_{i,j} (1 + f_x^2 + f_y^2)^{1/2} \Big|_{(x_i^*, y_j^*)} \Delta x \Delta y$$



$$SA = \iint_R \sqrt{1 + f_x^2 + f_y^2}\, dA$$

e.g.

$$z = \sqrt{R^2 - x^2 - y^2}$$

$$f_x = \frac{-x}{\sqrt{R^2 - x^2 - y^2}}$$

$$f_x^2 = \frac{x^2}{R^2 - (x^2 + y^2)}$$

$$f_y^2 = \frac{y^2}{R^2 - (x^2 + y^2)}$$

$$1 + f_x^2 + f_y^2 = \frac{R^2}{R^2 - x^2 - y^2}$$

$$\iint_R \frac{R}{\sqrt{R^2 - x^2 - y^2}} \, dA \qquad \int_0^{2\pi} \int_0^1 \frac{R}{\sqrt{R^2 - r^2}} \, r \, dr \, d\theta$$

$$u = R^2 - r^2$$

$$du = -2r\,dr$$

$$\int_0^R \frac{R}{\sqrt{R^2 - r^2}} \, r \, dr = \int_{R^2}^{0} -\frac{1}{2} \frac{R}{\sqrt{u}} \, du$$

$$= \frac{R}{2} \int_0^{R^2} \frac{1}{\sqrt{u}} \, du$$

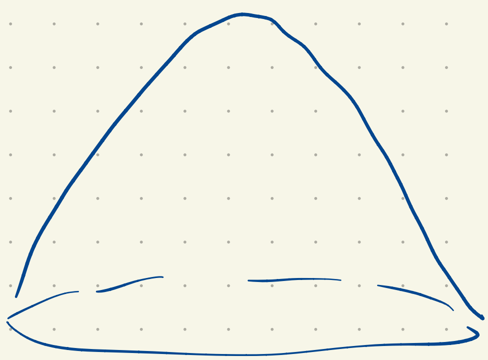
$$= R u^{1/2} \Big|_0^{R^2}$$

$$= R^2$$

$$\int_0^{2\pi} 1 \, d\theta = \boxed{2\pi R^2}$$

Whole sphere: $4\pi \cdot R^2$

e.g.



$$z = 4 - x^2 - y^2$$

$$f_x = -2x$$

$$f_y = -2y$$

$$\iint_R \sqrt{1+4x^2+4y^2} \, dA = \int_0^{2\pi} \int_0^2 \sqrt{1+4r^2} \, r \, dr \, d\theta$$

$$u = 1+4r^2$$

$$du = 8r \, dr$$

$$\vec{F}(x,y) = \langle x, y, f(x,y) \rangle$$

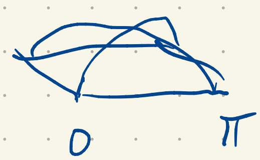
$$\vec{F}_x = \langle 1, 0, f_x \rangle$$

$$\vec{F}_y = \langle 0, 1, f_y \rangle$$

$$\vec{F}_x \times \vec{F}_y = \langle -f_x, -f_y, 1 \rangle$$

etc.

$\sin(x)\sin(y)$



$$\sqrt{1 + \cos^2(x)\sin^2(y) + \sin^2(x)\cos^2(y)}$$

Have fun!

≈ 12.05

$f = @(x,y)\ \ 1 + c^2(x)\,\cos^2(y)\ \ldots$