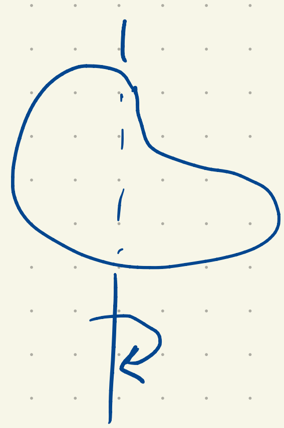
Last class

mass $m = \iint\limits_R \rho(x,y)\,dA$

1st moments $M_y = \iint\limits_R x\,\rho(x,y)\,dA$

$$M_x = \iint\limits_R y\,\rho(x,y)\,dA$$

gM_y is torque



$\bar{x} = \frac{M_y}{m}$ $\bar{y} = \frac{M_x}{m}$ $(\bar{x}, \bar{y})$ center of mass

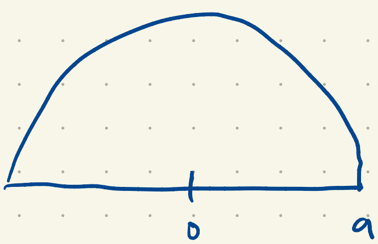
2^{nd} moments (moments of Inertia)

$$I_y = \iint_R x^2 \rho(x,y)\, dA$$

resistance to rotation due to mass

$$I_x = \iint_R y^2 \rho(x,y)\, dA$$

$$I_0 = \iint (x^2+y^2)\, \rho(x,y)\, dA$$



find $\bar{y}$ if density is proportional to distance from origin

$$\rho(x,y) = k\sqrt{x^2+y^2}$$

$$\iint_R k y \sqrt{x^2+y^2}\, dA = \int_0^{\pi}\int_0^{a} k\, r \sin\theta\, r\, r\, dr\, d\theta$$

$$= \int_0^{\pi}\int_0^{a} k\, r^3 \cos\theta \, dr\, d\theta$$

$$= k \int_0^{\pi} \frac{r^4}{4}\Big|_0^{a} \cos\theta \, d\theta$$

$$= \frac{ka^4}{4} \; - \cancel{\cos\theta} \Big|_0^{\pi}$$

$$= \frac{ka^4}{2}$$

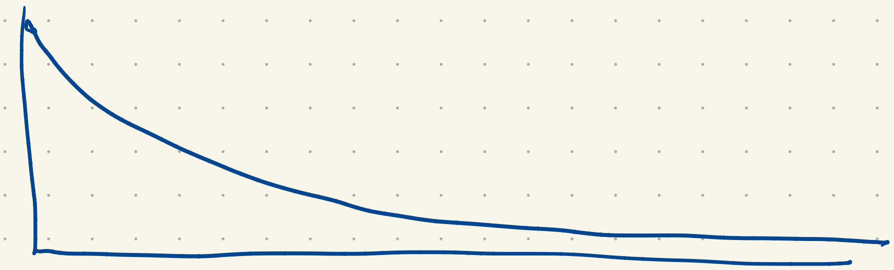
$$m = \iint_R k\sqrt{x^2 + y^2} = \int_0^{\pi}\int_0^{a} k\, r^2 \, dr\, d\theta$$

$$= \frac{ka^3}{3}\pi$$

$$\bar{y} = \frac{M_x}{m} = \frac{ka^4}{2} \cdot \frac{3}{ka^3\pi} = \frac{3}{2\pi} a$$

Probability

Waiting



$p(t) = a e^{-at}$ $\uparrow$ pdf

$$\int_0^\infty a e^{-at}\,dt = \int_0^\infty e^{-u}\,du$$

$$= -e^{-u}\Big|_0^\infty = 1$$

You'll get your coffee

$$\int_{t_0}^{t_1} a e^{-at}\,dt$$: probability the wait is between t_0 and t_1

Expected wait $\bar{t}$

$$\int_0^\infty t\, a e^{-at}\,dt$$

$u = t \qquad du = dt$

$v = e^{-at} \qquad dv = -a e^{-at}\,dt$

$$t e^{-at}\Big|_0^{\infty} - \int_0^{\infty} e^{-at}\,dt$$

$$0 - \frac{(-1)}{a} e^{-at}\Big|_0^{\infty} = \frac{1}{a}$$

This works in two dimensions.

$$p(x,y) \geq 0 \qquad \iint_{R} p(x,y)\,dA = 1$$

pdf (joint)

$$\iint_{D} p(x,y)\,dA$$ probability event lies in R.

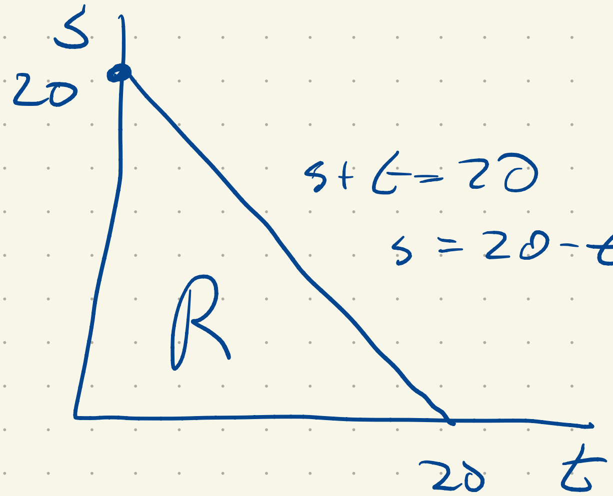
e.g. wait times for ticket: $10e^{-t/10}$

wait times for popcorn: $5e^{-s/\cancel{5}}$

independent

$$p(t,s) = \frac{1}{50} e^{-t/10} e^{-s/5}$$

What is the probability of waiting < 20 minutes for both ticket and popcorn?



$s + t \leq 20$

$$\frac{1}{50} \iint_R e^{-t/10} e^{-s/5}\, dA = \frac{1}{50} \int_0^{20} \int_0^{20-t} e^{-t/10} e^{-s/5}\, ds\, dt$$

$$= \frac{1}{50} \int_0^{20} e^{-t/10} \left(\frac{-1}{5} e^{-s/5} \right) \Big|_0^{20-t} dt$$

$$= \frac{1}{10} \int_0^{20} e^{-t/10} \left[-e^{t/5 - \frac{20}{5}} + 1 \right] dt$$

$$= \frac{1}{10} \int_0^{20} e^{-t/10} - e^{+t/10 - \frac{20}{5}} \, dt$$

$$= \frac{1}{10} \int_0^{20} e^{-t/10} - e^{-4} e^{+t/10} \, dt$$

$$= \frac{1}{10} \left(-10 e^{-t/10} - e^{-4} \, 10 e^{t/10} \right) \Big|_0^{20}$$

$$= -\left(e^{-t/10} + e^{-4} e^{t/10} \right) \Big|_0^{20}$$

$$= -\left(e^{-2} + e^{-4} e^{2} \right) + \left(1 + e^{-4} \right)$$

$$\approx 0.75.$$