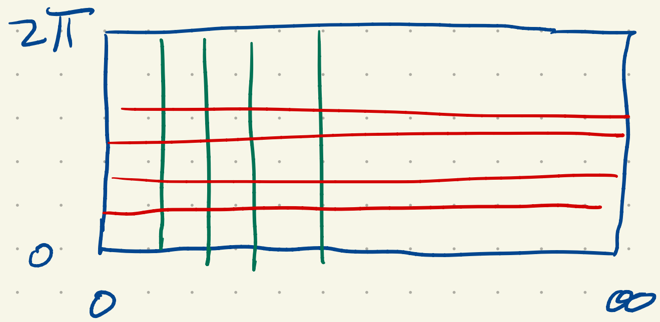
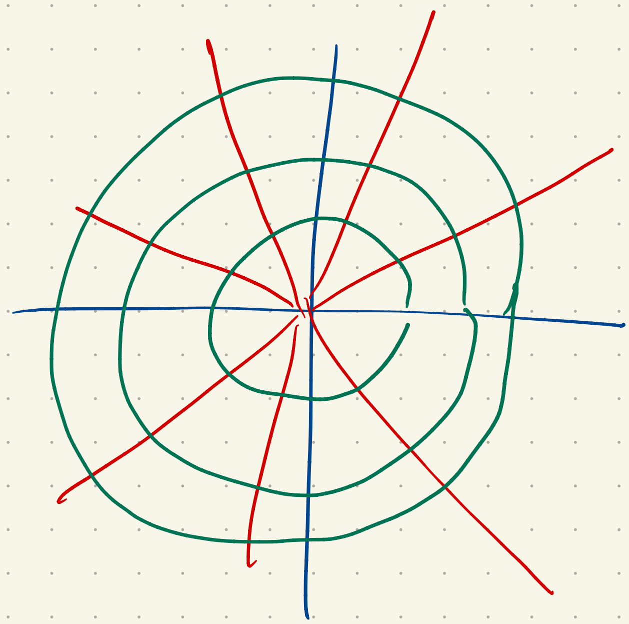
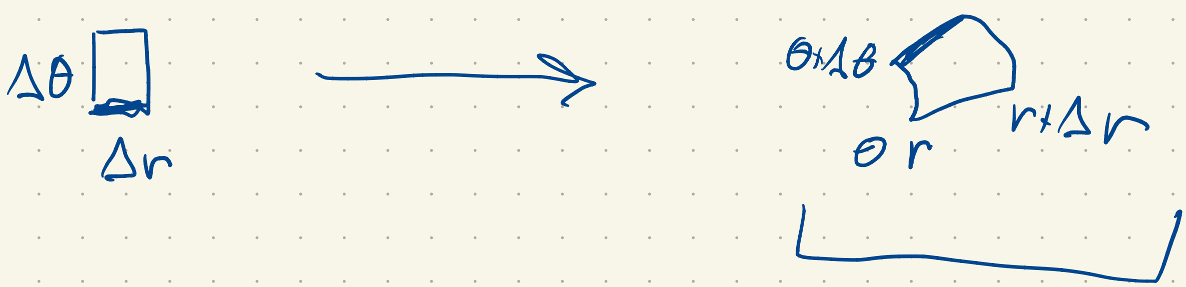
Section 15.3

Polar coordinates



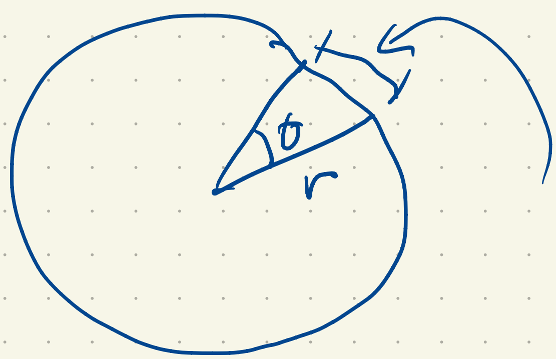




like a little rectangle

length Δr.

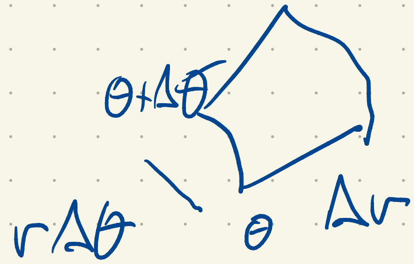
width? depends on r



total circumference $2\pi r$

part with angle θ

$$\frac{\theta}{2\pi} 2\pi r = \theta r$$

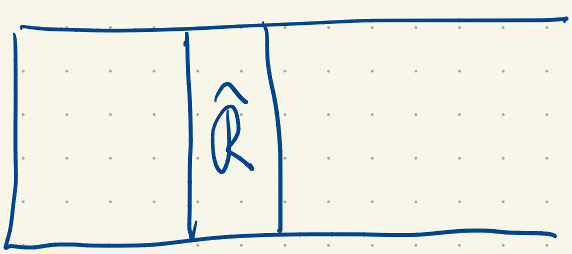


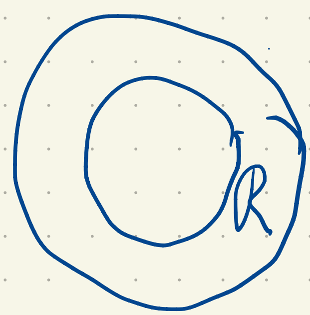
area is approximately

$r\Delta r\Delta\theta$

You want to integrate

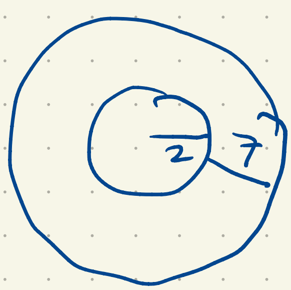
$$\iint_R f(x,y)\,dA$$





$$\iint_{\hat{R}} f(r\cos\theta, r\sin\theta)\underbrace{d\hat{A}}_{r\,dr\,d\theta} \qquad \iint_R f(x,y)\underbrace{dA}_{dx\,dy}$$

e.g.



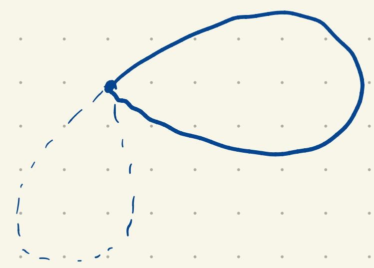
$$\int_0^{2\pi}\int_2^7 1\; r\,dr\,d\theta = \int_0^{2\pi} \frac{r^2}{2}\Big|_2^7$$

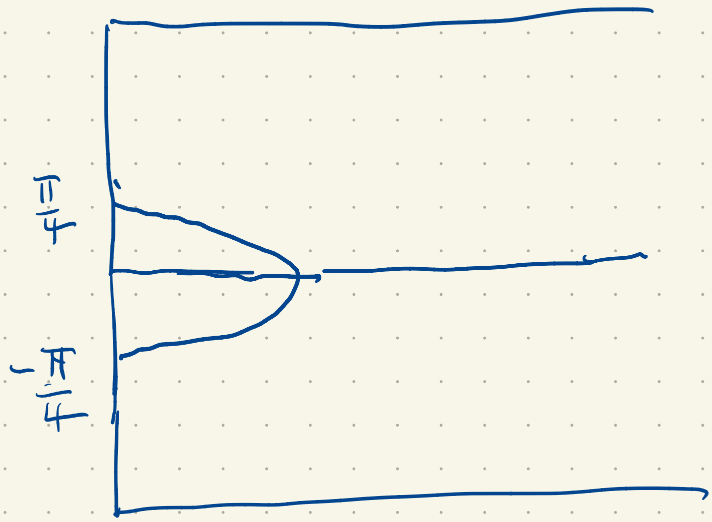
$$= \pi(7^2-2^2)$$

$$= \pi 45$$

e.g.

$r = \cos 2\theta$





$$\int_{-\frac{\pi}{4}}^{\pi/4} \int_0^{\cos(2\theta)} 1 \, r \, dr \, d\theta$$

$$\int_{-\frac{\pi}{4}}^{\pi/4} \frac{r^2}{2} \Big|_0^{\cos(2\theta)} d\theta$$

$$\int_{-\frac{\pi}{4}}^{\pi/4} \frac{\cos^2(2\theta)}{2} \, d\theta$$

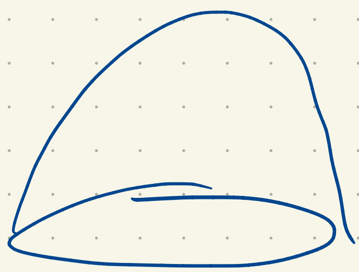
$$\int_{-\frac{\pi}{4}}^{\pi/4} \frac{(1+\cos(4\theta))}{4} \, d\theta$$

$$\cos^2(\theta) = \frac{1+\cos(2\theta)}{2}$$

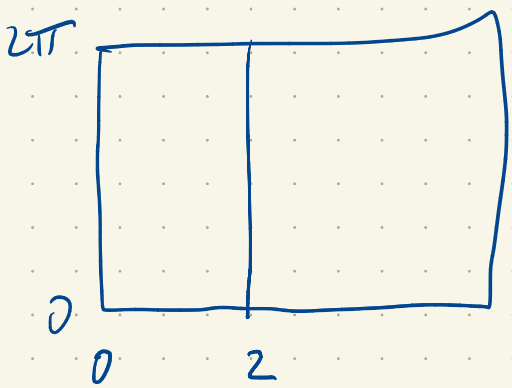
$$\frac{1}{4}\left[\theta + \frac{\sin(4\theta)}{4}\right]_{-\frac{\pi}{4}}^{\pi/4}$$

$= \pi/8$

$4 - x^2 - y^2$



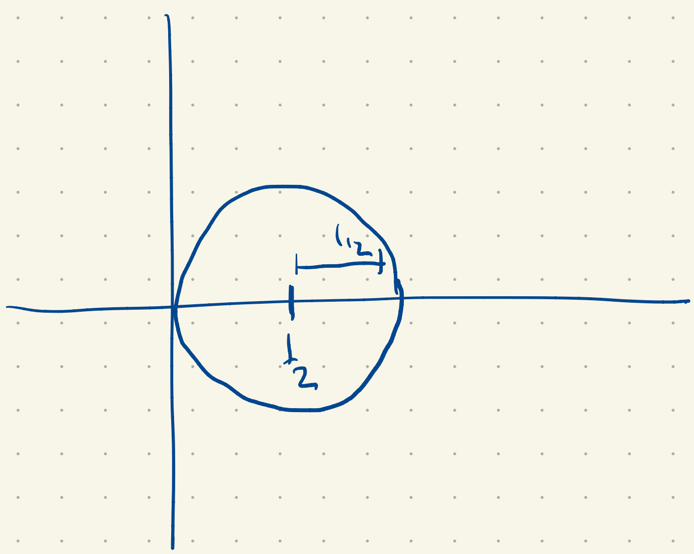
$$\int_0^{2\pi} \int_0^2 (4 - r^2)\, r\, dr\, d\theta$$



$$\int_0^{2\pi} \left(4\frac{r^2}{2} - \frac{r^4}{4}\right)\Big|_0^2 d\theta$$

$$\int_0^{2\pi} 2 \cdot 2^2 - \frac{4^2}{4}\, d\theta$$

$$\int_0^{2\pi} 8 - 4\, d\theta = \boxed{8\pi}$$



$r = \cos\theta \qquad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$

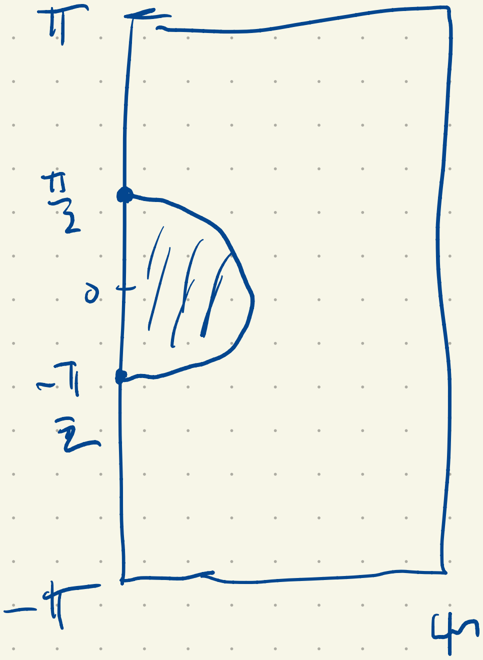
$r^2 = r\cos\theta$

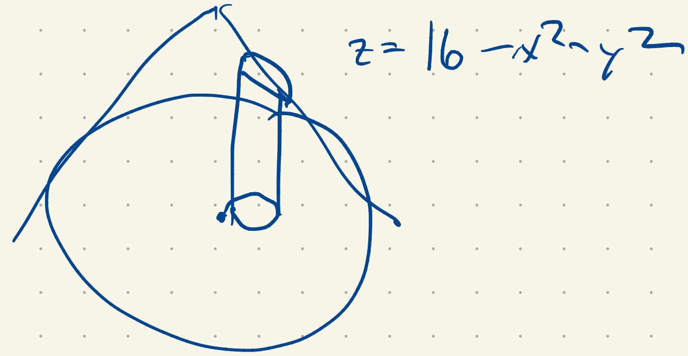
$x^2 + y^2 = x$

$x^2 - x + y^2 = 0$

$x^2 - x + \frac{1}{4} + y^2 = \frac{1}{4}$

$\left(x - \frac{1}{2}\right)^2 + y^2 = \frac{1}{4}$





$$\int_{-\frac{\pi}{2}}^{\pi/2} \int_0^{\cos\theta} (16 - r^2)\, r\, dr\, d\theta$$

$$\int_{-\frac{\pi}{2}}^{\pi/2} \left(\frac{16 r^2}{2} - \frac{r^3}{3} \right) \Big|_0^{\cos\theta} d\theta$$

$$\int_{-\pi/2}^{\pi/2} \left(8\cos^2\theta - \frac{\cos^3\theta}{3} \right) d\theta$$

$$\int_{-\pi/2}^{\pi/2} 8\left[\frac{1 + \cos 2\theta}{2} \right] - \frac{1}{3}\left(1 - \sin^2\theta\right)\cos\theta\, d\theta$$

$$\int_{-\pi/2}^{\pi/2} 4 + 4\cos 2\theta - \frac{1}{3}\cos\theta + \frac{1}{3}\sin^2\theta\cos\theta\, d\theta$$

$$4\theta + \frac{4\sin(2\theta)}{2} - \frac{1}{3}\sin\theta + \frac{\sin^3\theta}{9} \Bigg|_{-\pi/2}^{\pi/2}$$

$$2\left[4\frac{\pi}{2} + 2\sin(\pi) - \frac{1}{3}\sin(\pi/2) + \frac{\sin^3(\pi/2)}{9}\right]$$

$$2 \cdot \left[2\pi - \frac{1}{3} + \frac{1}{9}\right] = 2 \cdot \left[2\pi - \frac{2}{9}\right]$$

$$= 4\pi - \frac{4}{9}$$ whew!