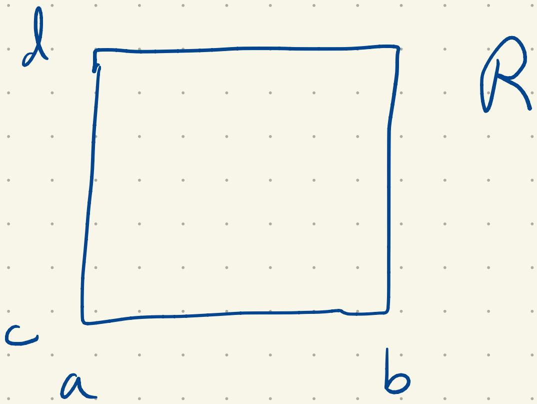
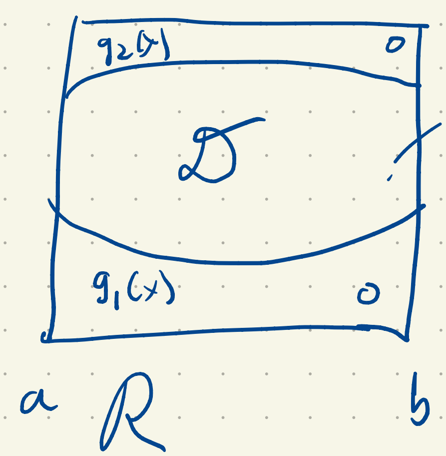
Section 15.2



$$\iint_R f(x,y)\,dA = \int_c^d \int_a^b f(x,y)\,dx\,dy = \int_a^b \int_c^d f(x,y)\,dy\,dx$$

If f is cts.

In fact it's true for a broader class of nice functions. Here are some

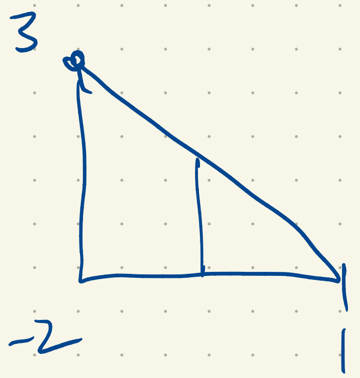


$$\iint_D f(x,y)\,dA = \iint_R f(x,y)\,dA$$

$$\int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\,dy\,dx$$

e.g. integrate $f(x,y) = 4-y$ in region bounded by

$x=-2$, $y=0$, $y=1-x$



$$\int_{-2}^{1}\int_0^{1-x} 4-y \,dy\,dx$$

$$= \int_{-2}^{1} \left(4y-\frac{y^2}{2}\right)\Bigg|_0^{(1-x)} dx$$

$$= \int_{-2}^{1} 4(1-x) - \frac{(1-x)^2}{2}\,dx$$

$$= 4x - \frac{4x^2}{2} + \frac{(1-x)^3}{6}\Bigg|_{-2}^{1}$$

$$= 4-2-\left[-8-8+\frac{27}{6}\right]$$

$$= 2 + 8 + 8 - \frac{9}{2}$$

$$= 18 - 4 - \frac{1}{2} = 14 - \frac{1}{2} = \frac{27}{2}$$

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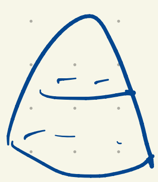
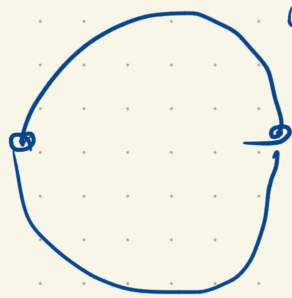
e.g. $z = 4 - x^2 - y^2$

Find region bounded by above and $z = 0$



$y = \sqrt{4-x^2}$



$x^2 + y^2 = 4$

$y = \pm\sqrt{4-x^2}$

$y = -\sqrt{4-x^2}$

$$\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} 4 - x^2 - y^2 \, dy \, dx$$

$$\int_{-2}^{2}\left[\left(4y - x^2y - \frac{y^3}{3}\right)\Bigg|_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\right]dx$$

$$\int_{-2}^{2}\left[(4-x^2)\,2\sqrt{4-x^2} - \frac{2\left(\sqrt{4-x^2}\right)^3}{3}\right]dx$$

$$\int_{-2}^{2} \frac{4}{3}(4-x^2)\sqrt{4-x^2}\,dx$$

$$x = 2\sin\theta$$
$$dx = 2\cos\theta\, d\theta$$

$$\frac{4}{3}\int_{-2}^{2}(4-x^2)^{3/2}\,dx$$

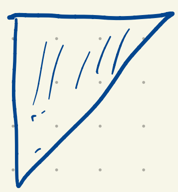
$$\frac{4}{3}\int_{-\pi/2}^{\pi/2} 4^{3/2}\left(\cos^2\theta\right)^{3/2} 2\cos\theta\, d\theta$$

$$\frac{64}{3}\int_{-\pi/2}^{\pi/2} \cos^4\theta \, d\theta$$

$$= \frac{64}{3}\left[\frac{1}{4}\cos^3 x \sin x + \frac{3}{8}\cos x \sin x + \frac{3}{8}x\right]_{-\pi/2}^{\pi/2}$$

$$= 8\pi \quad (!)$$

$$\int_0^1 \int_x^1 \sin(y^2)\, dy \, dx$$



$$\int_0^1 \int_0^y \sin(y^2)\, dx \, dy = \int_0^1 y \sin(y^2)\, dy$$

$$u = y^2$$

$$du = 2y\, dy$$

$$= \frac{1}{2}\int_0^1 \sin(u)\, du$$

$$= \frac{1}{2} \left(-\cos(u)\right)\Big|_0^1$$

$$= \frac{1}{2} \left[-\cos(1) + \cos(0)\right]$$

$$= \frac{1}{2} \left[1 - \cos(1)\right]$$