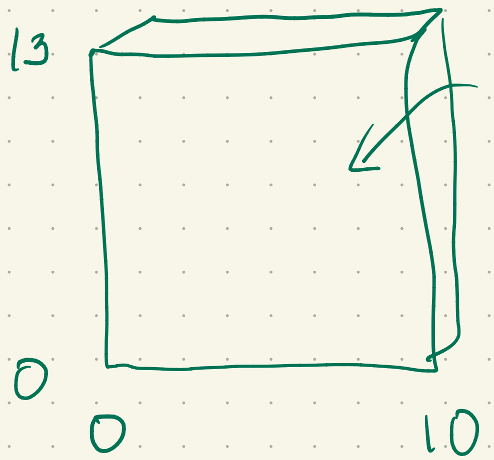
Integration

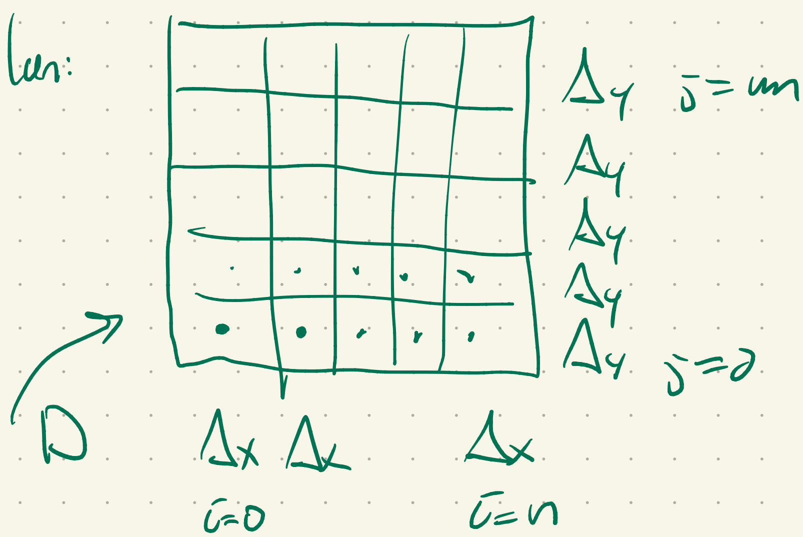
thin plate



$\rho(x,y) = \left(6 + \frac{x}{10} - \frac{y}{13}\right)\ g/cm^2$

What is the mass of the plate?

Plan:

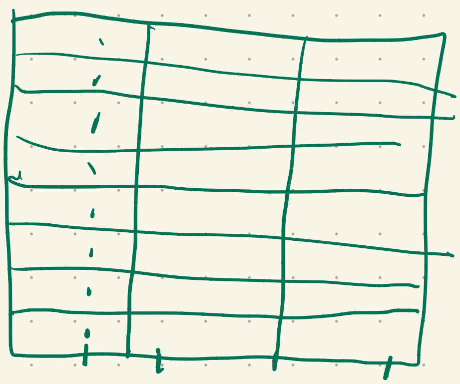


In square (i,j) pick $P_{i,j} = (x_i, y_j)$

Approximate density: $\sum_{j=1}^{m}\sum_{i=1}^{n} \rho(P_{i,j})\,\Delta x\,\Delta y$

The thing that you get is

$$\iint_D \rho(x,y)\,dx\,dy$$



$$\sum_{j=1}^{m} \sum_{i=1}^{3} \rho(x_i, y_j)\,\Delta_x\,\Delta_y$$

e.g. $x_1 = 2,\ x_2 = 5,\ x_3 = 7$

$$\left[\sum_{j=1}^{m} \rho(2, y_j)\,\Delta_y\right]\Delta x + \quad \cdots + \qquad \Delta x$$

As $m \to \infty$ $\quad \int_0^{13} \rho(2,y)\,dy\,\Delta x + \int_0^{13} \rho(5,y)\,dy\,\Delta x + \int_0^{10} \rho(7,y) =$

$$\sum_{i=1}^{n}\left[\int_0^{13}\rho(x_i,y)\,dy\right]\Delta x$$

$$f(x)=\int_0^{13}\rho(x,y)\,dy$$

$$\sum_{i=1}^{n} f(x_i)\Delta x$$

As $n\to\infty$
$$\int_0^{10}\int_0^{13}\rho(x,y)\,dy\,dx$$

Could have done it in other order

$$\int_0^{13}\int_0^{10}\rho(x,y)\,dx\,dy$$

These are called iterated integrals.

Thm: Fubini

For a continuous function $f(x,y)$ on a rectangle

$R=[a,b]\times[c,d]$

$$\iint_R f(x,y)\,dx\,dy=\int_a^b\int_c^d f(x,y)\,dy\,dx=\int_c^d\int_a^b f(x,y)\,dx\,dy.$$

$$\left(6 + \frac{x}{10} - \frac{2y}{13}\right) g/cm^2$$

$$\int_0^{13}\int_0^{10} 6 + \frac{x}{10} - \frac{2y}{13}\, dx\, dy = \int_0^{13} 6x + \frac{x^2}{20} - \frac{2xy}{13}\Big|_0^{10} dy$$

$$= \int_0^{13} 60 + \frac{100}{20} - \frac{20y}{13}\, dy$$

$$= 60y + 5y - \frac{20y^2}{2\cdot 13}\Big|_0^{13}$$

$$= 60\cdot 13 + 5\cdot 13 - 10\cdot 13$$

$$= 60\cdot 13 - 5\cdot 13$$

$$= 55\cdot 13$$

$$\begin{array}{r} {}^{1} \\ 550 \\ \underline{165} \\ 715\,g \end{array}$$

What is the average density of the plate?

Area: $10 \cdot 13 = 130 \text{ cm}^2$

$$\frac{715 g}{130 \text{ cm}^2} =$$

In general, avg value of $f(x,y)$ on R is

$$\frac{1}{\text{Area}(R)} \iint_R f(x,y)\,dx\,dy.$$