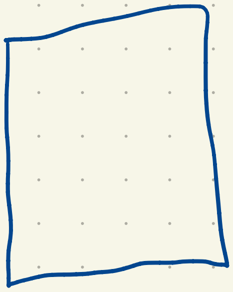

closed bounded domain.

(includes boundary)

fits in a box.

A continuous function on such a domain will attain a max/min.

This happens either at

1) an interior critical point

2) on the boundary.

E.g. Maximize $V = xyz$ subject to $x, y, z \geq 0$

$$x+y+z \leq 96$$

$$z \leq 96 - x - y$$

(skipping negs!)

$$z = 96 - x - y$$

$x \geq 0$

$y \geq 0$

$x + y \leq 96$

$$V = xy(96 - x - y)$$

$$V_x = y(96 - x - y) - xy$$

$$= y(96 - y) - 2xy = y[96 - y - 2x]$$


$$V_y = x(96 - x) - 2xy = x[96 - x - 2y]$$

$V_x = 0$: $y = 0$ or $96 - y - 2x = 0$

$V_y = 0$: $x = 0$ or $96 - x - 2y = 0$

$(0,0)$, $(0,96)$ $(96,0)$

$$-y + x - 2x + 2y = 0$$

$$-x + y = 0 \quad x = y$$

$$96 - x - 2x = 96 - 3x$$

$$x = 32$$

$$y = 32$$

$V_{xx} = -2y$ $V_{yy} = -2x$ $V_{xy} = 96 - 2y - 2x$

$$D = V_{xx} V_{yy} - (V_{xy})^2 = 4xy - (96 - 2x - 2y)^2$$

$D(32,32) = 3072 > 0$

$V_{xx} < 0 \Rightarrow$ local max

Whew!

$z = 96 - 32 - 32 = 32$ (!)

Cube: $32^3$

Last class:

critical point: $\vec{\nabla} f = 0$ or DNE.

At a local min/max in interior of domain,
we have a crit point.

So if looking for max/min, in interior
need only look at critical points.

for $f(x,y)$ (2-d) we have a 2nd deriv test

$$H = \begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{bmatrix} \qquad D = |H| = f_{xx} f_{yy} - (f_{xy})^2$$

If $D > 0 \Rightarrow$ local min/max

$D < 0 \Rightarrow$ saddle

$D = 0 \Rightarrow$ inconclusive

$f_{xx} > 0 \Rightarrow$ local min ($f_{yy}$ also)
$f_{xx} < 0 \Rightarrow$ local max