direction: $(-\hat{i} - 2\hat{j} + 6\hat{k})$

or normalize

Claim: the gradient is perpendicular to the level surfaces of a function

$F(x,y,z) = c$

$x^2+y^2+z^2=c$

level surface

$\vec{r}(t)$ lives entirely in $F(x,y,z)=c$

$$F(\vec{r}(t))=c$$

$$\frac{d}{dt}F(\vec{r}(t))=0$$

$$F_x\dot{x}+F_y\dot{y}+F_z\dot{z}=0$$

$$\vec{\nabla}F\cdot\vec{r}\,'=0$$

So we define the tangent plane to level surf at (x_0,y_0,z_0) by

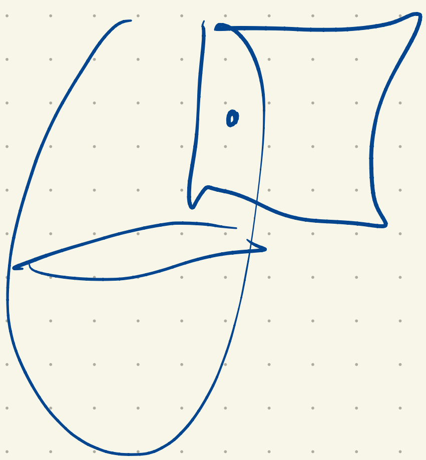
$$F_x(x-x_0)+F_y(y-y_0)+F_z(z-z_0)=0$$

e.g. $\frac{x^2}{4} + y^2 + \frac{z^2}{9} = 3$

$F(x,y,z) = \uparrow$ at $(-2,1,3)$

$F_x = -1 \qquad F_y = 2 \qquad F_z = \frac{2}{3}$

$-1(x+2) + 1\cdot(y-1) + 3(z-3) = 0$



3 facts from calc I

a) If $f(x)$ attains a local max/min on $[a,b]$ but not at an endpoint, $f'(x)=0$ there

b) If $f'(x)=0$ and $f''(x)>0 \Rightarrow$ local min

$f''(x)<0 \Rightarrow$ local max

local max's.

New versions

$f(x_0,y_0) \geq f(x,y)$

for all (x,y) close to (x_0,y_0).

a) $f(x,y)$

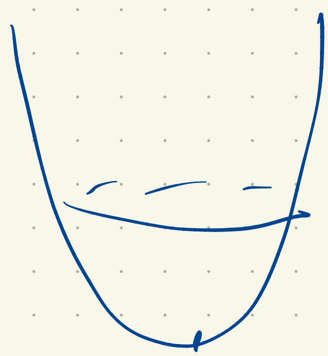
If we attain a local max/min at (x_0,y_0) in interior and derivatives exist there

$$\frac{\partial f}{\partial x}(x_0,y_0)=0 \qquad \frac{\partial f}{\partial y}(x_0,y_0)=0$$

$\left(\vec{\nabla} f(x_0, y_0) = \vec{0} \right)$

$f(x,y,z)$ $\left(\frac{\partial f}{\partial z} = 0 \text{ also} \right)$

We saw this



Critical point: $f_x = 0$ and $f_y = 0$

(or DNE)

E.g. $f(x,y) = xy(x-2)(y+3)$

Find crit pts

$f_x = y(x-2)(y+3) + xy(y+3) = y(y+3)[2x-2]$

$$f_y = x(x-2)(y+3) + xy(x-2) = x(x-2)[2y+3]$$

$f_x = 0$ $y=0$, $y=-3$, $x=1$ | $f_y = 0$ $x=0$, $x=2$, y

$y=0$ $x=0, 2$ $(1,0)$ $(0,0)$

$y=3$ $x=0, x=2$ $(0,3)$ $(2,3)$

$x=1$ $y=-3/2$ $(1,-3/2)$

So: there are 5 possible locations for min/max.

How can we determine what kind?

Models: x^2+y^2 $-x^2-y^2$ x^2-y^2

min   max   saddle

$$\begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{bmatrix}$$

Hessian matrix

symmetric

$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \quad |\cdot| = 4 > 0$$

diagonal elements are > 0

$$\begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} \quad |\cdot| = 4 > 0$$

diagonal elements are < 0

$$\begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix} \quad |\cdot| = -4 < 0$$

$$D = f_{xx} f_{yy} - (f_{xy})^2 = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}$$

$D > 0$, diag $> 0 \Rightarrow$ local min

, diag $< 0 \Rightarrow$ local max

$D < 0 \Rightarrow$ saddle point

$D = 0$ inconclusive (like $f''(x) = 0$).

$$f_x = y(x-2)(y+3) + xy(y+3) = y(y+3)[2x-2]$$

$$f_y = x(x-2)(y+3) + xy(x-2) = x(x-2)[2y+3]$$

$$f_{xx} = 2(y)(y+3) \qquad f_{xy} = 2(x-1)[2y+3]$$

$$f_{yy} = 2(x)(x-2)$$

$$D = 4xy(x-2)(y+3) - 4(x-1)^2(2y+3)^2$$

$$D\left(1,-\tfrac{3}{2}\right) = 1\cdot(-3)(1-2)\cdot(-3+6) - 4\cdot 0$$

$$= 9 > 0 \qquad f_{yy} = -2 < 0$$

$\Rightarrow$ local max

$$D(0,0) = -4\cdot 1\cdot 9 = -36$$

$\Rightarrow$ saddle