Last class:

- $D_v f(x_0, y_0)$

  Rate of change of f at (x_0, y_0)

  if moving with velocity $\vec{v}$.

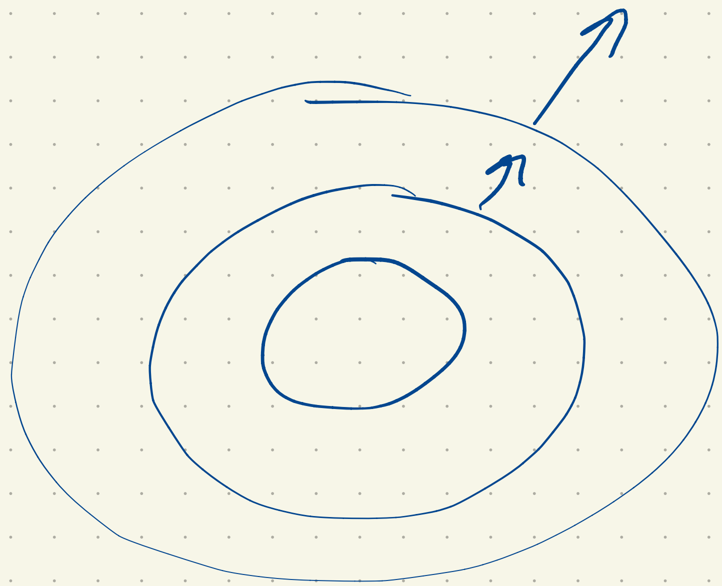
  Book requires $\vec{v}$ be a unit vector.

- $\vec{\nabla} f = \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \rangle$, gradient vector

$$D_v f = \vec{\nabla} f \cdot \vec{v}$$

$$f(x,y) = \frac{1}{2}(x^2+y^2)$$

$$\vec{\nabla} f = x\hat{i} + y\hat{j}$$



$\vec{v} \cdot \vec{\nabla} f = 0$ function not changing instantaneously in $\vec{v}$ direction

Q: When $\vec{u}$ is a unit vector, when is $D_{\vec{u}} f(x_0, y_0)$ at a maximum?

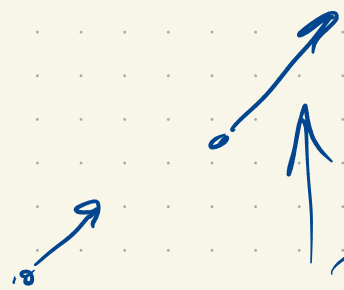
$$\vec{u} \cdot \vec{\nabla} f(x_0,y_0) = |\vec{u}| \cdot |\vec{\nabla} f(x_0,y_0)| \cdot \cos\theta$$

$$= |\vec{\nabla} f(x_0,y_0)| \cos\theta$$

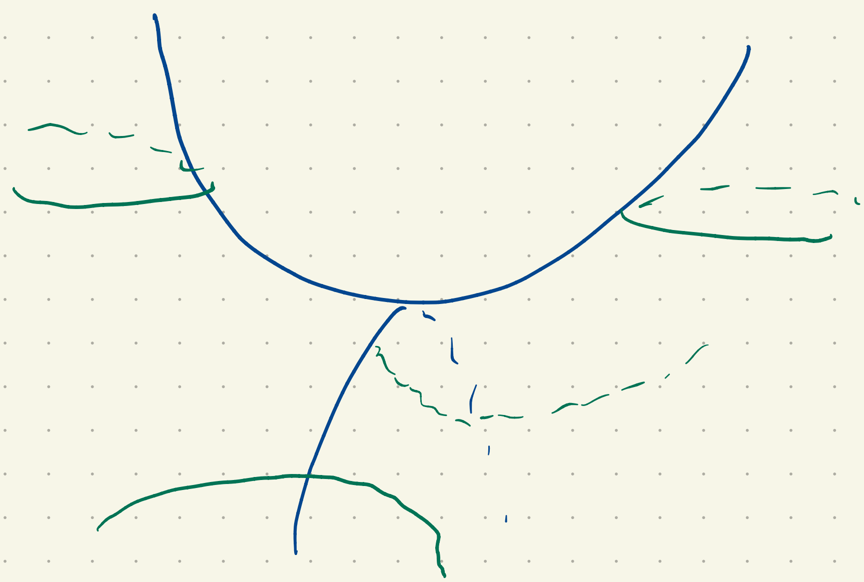
Max: $\theta = 1$, min $\theta = \pi$, 0 $\theta = \frac{\pi}{2}$

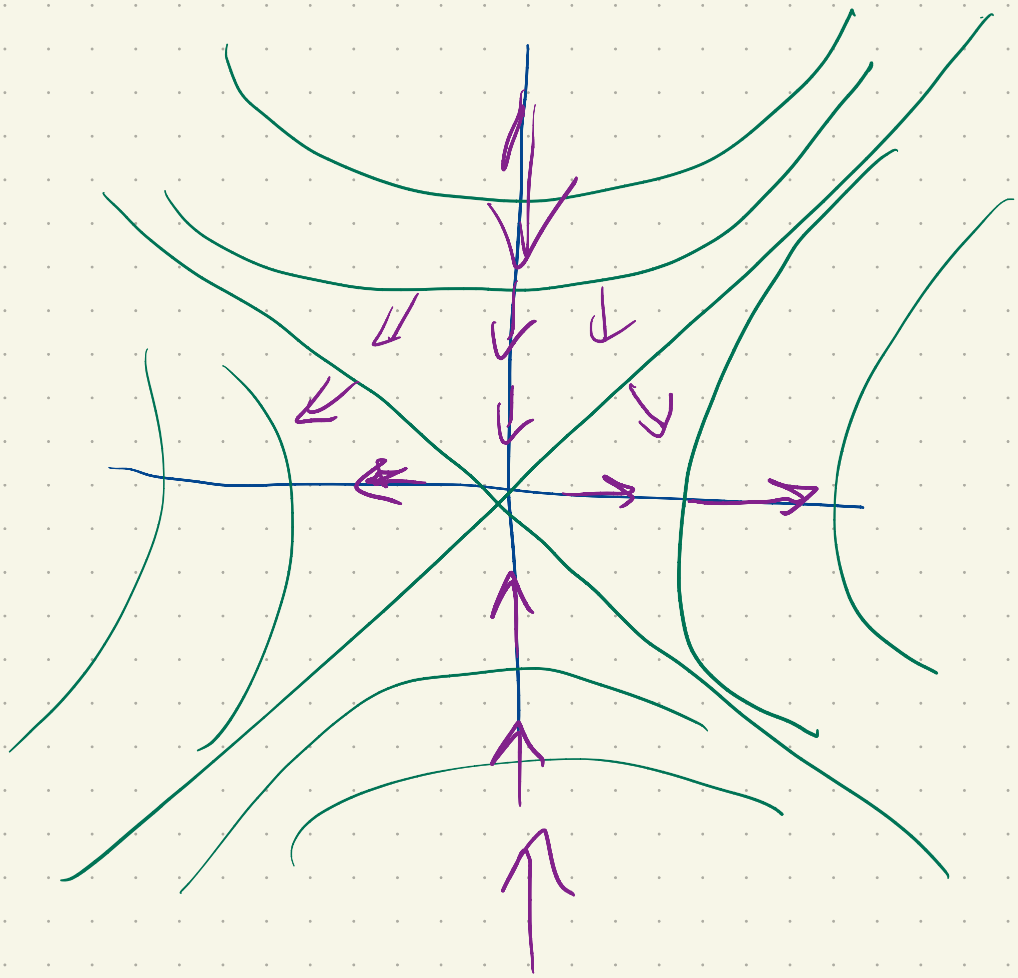
The direction of the gradient tells you the direction of travel that increases the function value the most.

The length tells you the rate of change in that direction



means function is steeper





Now 3-d

e.g.

$$T(x,y,z) = \frac{80}{1+x^2+2y^2+3z^2}$$

$$p = (1,1,-2)$$

in what direction is max increase?

$$\nabla T = T_x \hat{i} + T_y \hat{j} + T_z \hat{k}$$

$$= \frac{-160}{1+x^2+2y^2+3z^2}\left(-x\hat{i} - 2y\hat{j} - 3z\hat{k}\right)$$

$(1,1,-2)$: $\quad \nabla T = \frac{5}{8}\left(-\hat{i} - 2\hat{j} + 6\hat{k}\right)$

$$|\nabla T| = \frac{5}{8}\sqrt{41}$$

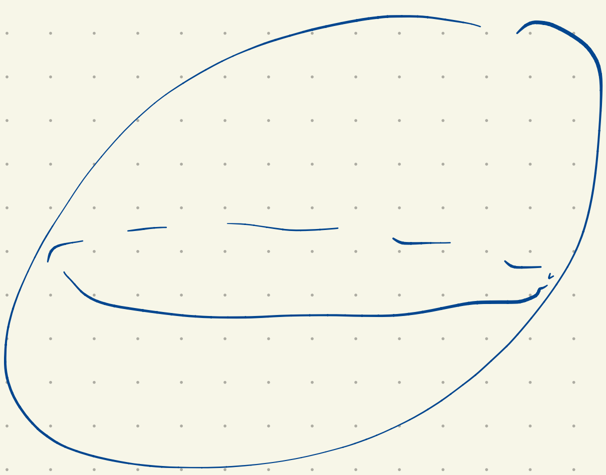
direction: $(-\hat{i} - 2\hat{j} + 6\hat{k})$

or normalize

Claim: the gradient is perpendicular to the level surfaces of a function

$F(x,y,z) = c$

$x^2+y^2+z^2=c$



level surface