Last class:

Linear approximation of $f(x,y)$ at (a,b)

$$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

$L(x,y) \approx f(x,y)$ for x, y near (a,b).

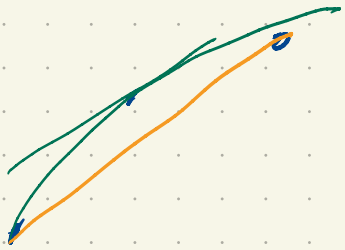
e.g. perceived temp as a function of temp, humidity

T \ Relative H	65	70	75
94	114	118	122
96	121	125	130
98	127	133	138

$I = f(T, H)$

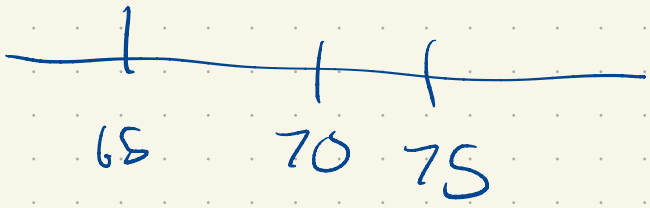
Approximate $f(97, 72)$.

We approximate the linear approximation!



$$\frac{\partial f}{\partial H}(96,70) \approx \frac{130-121}{10}$$

$$= 0.9$$



$$\frac{\partial f}{\partial T}(96,70) \approx \frac{133-118}{4}$$

$$= \frac{15}{4} = 3.75$$

$$f(96,70) = 125$$

$$L(T,H) \text{ "}=\text{"} \; 125 + 3.75(T-96) + 0.9(H-70)$$

$$L(97,72) = 125 + 3.75 + 1.8$$

$$= 130.55$$

Differentials

$$f(x,y) \approx f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

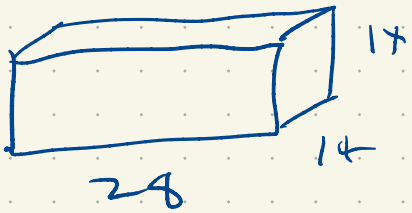
$$f(x,y) - f(a,b) \approx f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

$$df := f_x(x,y)\,dx + f_y(x,y)\,dy$$

Don't think about this too hard. You are just using the linear approx.

"If you change (x,y) to $(x+dx, y+dy)$ then f changes by an amount approximately df"

Eg: A cardboard box has outer dimensions 14, 14, 28 inches.



It is 1/8 inch thick.

Approximate the volume of cardboard.

$$V = xyz$$

$$dV = yz\,dx + xz\,dy + xy\,dz$$

$x = 14 \quad y = 14 \quad z = 28 \quad dx = dy = dz = \pm\frac{1}{8}$

$$dV = -14\cdot 28\cdot\frac{1}{4} - 14\cdot 28\cdot\frac{1}{4} - 14\cdot 14\cdot\frac{1}{4}$$

$$= -14^2\left[\frac{1}{2} + \frac{1}{2} + \frac{1}{4}\right]$$

$$= \frac{14^2\cdot 5}{4} = 7^2\cdot 5 = 245 \text{ m}^3$$

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e.g. A cone has ~~height~~ radius 10 cm and ~~radius~~ height 25 cm

with error $\pm$ 0.1 cm.

estimate the error in the volume.

$$V = \frac{1}{3}\pi r^2 h$$

$$dV = \frac{\partial V}{\partial r}\,dr + \frac{\partial V}{\partial h}\,dh$$

$$= \frac{2\pi r h}{3}\,dr + \frac{1}{3}\pi r^2\,dh$$

$$= \frac{2\pi \cdot 10.25}{3} \cdot \frac{1}{10} + \frac{1}{3}\pi \cdot 100 \cdot 0.1$$

$$= \frac{\pi}{3}\left[\frac{500}{10} + 10\right]$$

$$= \frac{\pi}{3} \cdot 60 = 20\pi \approx 60\ cm^3$$

A function is differentable if....