Newton 2:

$\vec{p}$: momentum (total quantity of motion)

$\vec{F}$: force

If object has mass m and velocity $\vec{v}$

$$\vec{p} = m\vec{v} = m\vec{r}\,'$$

The rate of change of momentum is force.

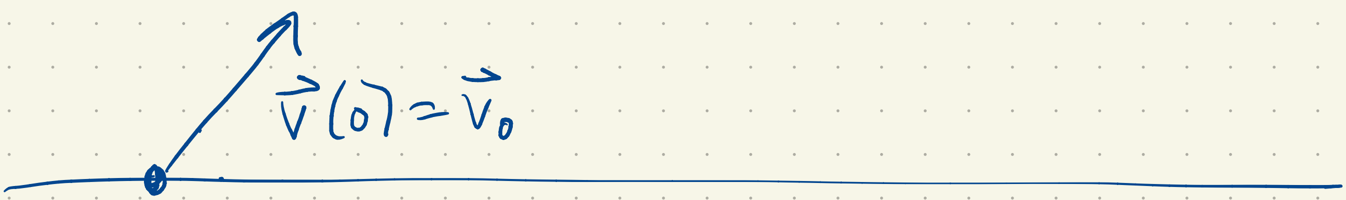
$$\frac{d}{dt}\vec{p} = \vec{F}$$

m constant: $\quad m\vec{r}\,'' = \vec{F} \qquad \left(\vec{F} = m\vec{a}\right)$

If you know the force acting on an object, you know the acceleration:

$$\vec{a} = \frac{1}{m}\vec{F}.$$

And if you know initial position and velocity then you can reconstruct the position,



$\vec{v}(0) = \vec{v}_0$

$\vec{r}(0) = \vec{r}_0$

Projectiles close to earth:

$$\vec{F}_g = -9.8\,\hat{k} \quad m/s^2$$

$$\vec{v}(t) = \int -9.8\,\hat{k}\,dt + \vec{C}_1$$

$$= -9.8t\,\hat{k} + \vec{C}_1$$

$$\vec{v}(0) = \vec{C}_1$$

$$\vec{v}(t) = -9.8t\,\hat{k} + \vec{v}_0$$

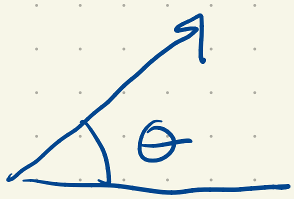
$$\vec{r}(t) = \int \vec{v}(t)\,dt + \vec{C}_2$$

$$\vec{r}(t) = -\frac{9.8t^2}{2}\hat{k} + \vec{v}_0 t + \vec{C}_2$$

$$\vec{r}(0) = 0 + 0 + \vec{C}_2$$

$$\vec{r}(t) = \vec{r}_0 + \vec{v}_0(t) - \frac{9.8}{2}t^2\hat{k}$$

($9.8 \to 0 \Rightarrow$ linear motion!)



$$\vec{r}_0 = \vec{0}$$

$$\vec{v}_0 = v_0\cos(\theta)\hat{i} + v_0\sin\theta\,\hat{k}$$

$$\vec{r}(t) = v_0\cos\theta\, t\,\hat{i} + \left[v_0\sin\theta\, t - \frac{9.8}{2}t^2\right]\hat{k}$$

$x = ct \qquad t = x/c$

$\alpha = \tan\theta$

$z = \alpha x - \beta x^2 \qquad \beta = \frac{9.8}{2}\frac{\sec^2\theta}{v_0^2}$

This is a parabolic trajectory.

When is $z = 0$? $\quad t\left[v_0 \sin\theta - \frac{9.8}{2} t\right] = 0$

$$t = 0 \quad \text{or}$$

$$t = \frac{2v_0}{9.8} \sin\theta$$

e.g. $v_0 = 150\,m/s \qquad \theta = \pi/4 = 45^\circ$

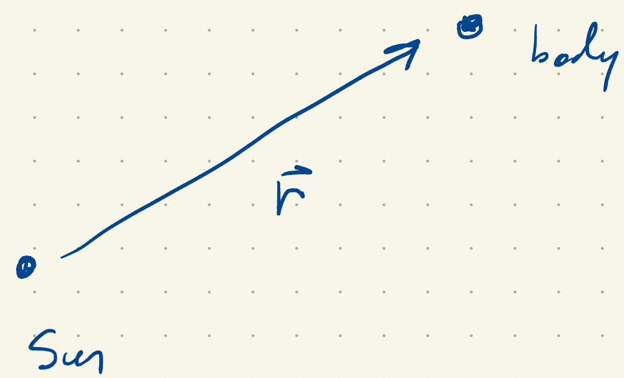
How far when strikes ground?

$$t = \frac{300}{9.8} \frac{1}{\sqrt{2}} \approx 21.64$$

$$x = v_0 \cos\theta\, t$$

$$= 150 \frac{1}{\sqrt{2}} \frac{300}{9.8} \frac{1}{\sqrt{2}} = \frac{150 \cdot 150}{9.8} \approx 2295\,m$$

which is nearly constant.



$$F_G = -\frac{G M_s m_b}{|\vec{r}|^2} \frac{\vec{r}}{|\vec{r}|}$$

Acceleration of body:

$$\frac{d}{dt}\left(m_b \vec{r}\,' \right) = - \frac{G M_s \vec{r}}{|\vec{r}|^3}$$

$$\vec{r}\,'' = - \frac{G M_s}{|\vec{r}|^3} \vec{r} \qquad M_s \to M$$

$\vec{p} = m_b \vec{r}\,' \rightarrow$ "linear momentum"

$$\vec{L} = \vec{r} \times \vec{p}$$ angular momentum about $\vec{O}$.

"total amount of angular motion"!

$$\frac{d}{dt}\vec{p} = \vec{F}_g \neq 0.$$ The body's momentum is not preserved.

But

$$\frac{d}{dt}\vec{L} = \frac{d}{dt}\vec{r}\times\vec{p} = \vec{r}\,'\times\vec{p} + \vec{r}\times\frac{d}{dt}\vec{p}$$

$$= \vec{r}\,'\times(m_b\vec{r}\,') + \vec{r}\times\vec{F}_g$$

$$= 0 + 0$$

$$(\vec{r}\,'\times\vec{r}\,'),\ (\vec{r}\times\vec{r})$$

Angular momentum ($\vec{L}$) about $\vec{O}$ is constant.

$$\frac{d}{dt}(\vec{r}-\vec{a})\times\vec{p} = -\vec{a}\times\vec{F}_g \neq 0 \quad \text{in general.}$$

Note $\vec{r} \perp \vec{L}$ so $\vec{r}$ is always contained in the plane thru origin perp to $\vec{L}$.

$$\vec{L} = L\hat{k} \qquad L > 0 \qquad \text{WLOG}$$

$$\vec{r} = r\cos(\theta)\,\hat{i} + r\sin\theta\,\hat{j}$$

$$\vec{r}\,' = r'\,\frac{\vec{r}}{r} + r\left(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j}\right)\theta'$$

$$\vec{r}\times\vec{r}\,' = \vec{r}\times r\left(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j}\right)\theta'$$

$$= r^2\left(c\,\hat{i} + s\,\hat{j}\right)\left(-s\,\hat{i} + c\,\hat{j}\right)\theta'$$

$$= r^2\left(c^2\,\hat{i}\times\hat{j} - s^2\,\hat{j}\times\hat{i}\right)\theta'$$

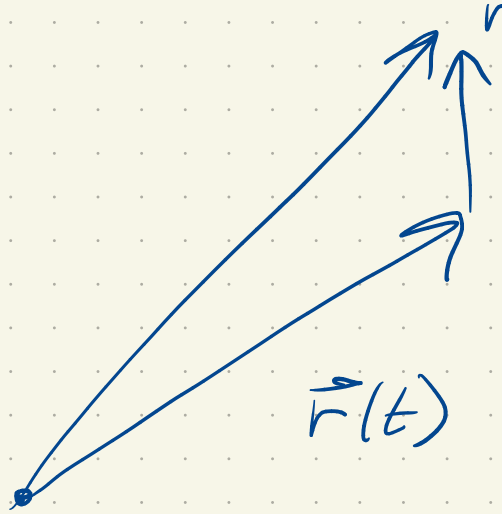
$$= r^2 (c^2 + s^2)\, \hat{\imath} \times \hat{\jmath}\, \theta'$$

$$= r^2 \theta' \hat{k}$$

$$L = |\vec{r} \times p| = m r^2 \theta'$$

$$\theta' = \frac{1}{m} \frac{L}{r^2}$$

Angle changes <u>faster</u> as $r \to 0$.



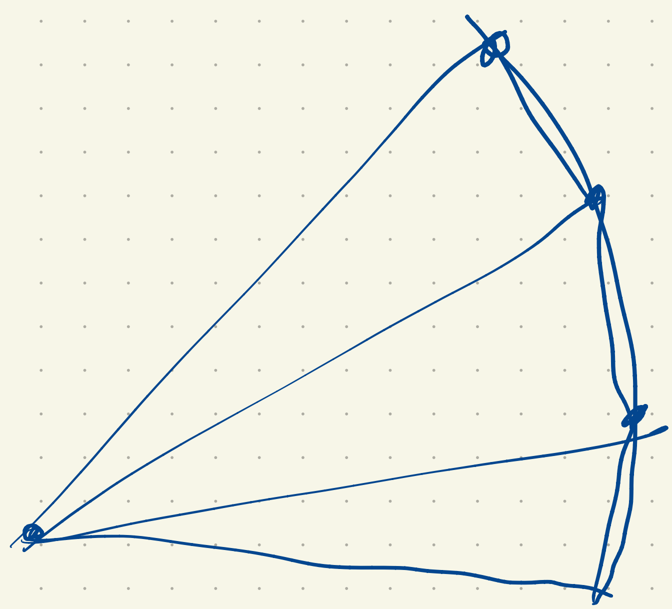
$$\frac{1}{2} \left| \vec{r} \times \vec{v} \Delta t \right| = \frac{1}{2} |\vec{r} \times \vec{p}| \frac{\Delta t}{m} = \frac{1}{2} \frac{L}{m} \Delta t$$

is the area of the triangle.

That is, in time Δt, the position vector sweeps out an area approximately $\frac{L}{2m} \Delta t$

Yep: equal $\Delta t \Longleftrightarrow$ equal area.

$$\frac{dA}{dt} = \frac{1}{2}\frac{L}{m}$$



Kepler 2: equal times $\Rightarrow$ equal area swept out.

is exactly conservation of angular momentum.