Section 13.4 (Acceleration, Velocity, Momentum, Force)

If $\vec{r}(t)$ describes position as a function of time

1) $\vec{v}(t) = \vec{r}\,'(t) = \frac{d}{dt}\vec{r}(t)$ is velocity

2) $|\vec{v}(t)| = |\vec{r}\,'(t)|$ is speed

3) $\vec{a}(t) = \vec{v}\,'(t) = \frac{d}{dt}\vec{v}(t) = \vec{r}\,''(t)$ is acceleration

e.g. If $\vec{r}(t) = \langle \sin(2t), \tan(t), 1-t \rangle \quad -\frac{\pi}{2} < t < \frac{\pi}{2}$

$$\vec{v}(t) = \langle 2\cos(2t), \sec^2(t), -1 \rangle$$

$$\vec{a}(t) = \langle -4\sin(2t), 2\sec(t)\sec(t)\tan(t), 0 \rangle$$

$$|\vec{r}\,'(t)| = \sqrt{4\cos^2(2t) + \sec^4(t) + 1}$$

e.g. Suppose a particle has

acceleration

$$\vec{a}(t) = \langle -\cos(t), -\sin(t), -1 \rangle$$

and $\vec{r}(0) = \langle 5, 2, 2 \rangle$

$\vec{r}'(0) = \langle 0, 1, 3 \rangle.$

Determine $\vec{r}(t)$.

$$\vec{v}'(t) = \vec{a}(t)$$

$$\vec{v}(t) = \int \vec{a}(t)\,dt + \vec{C}$$

$$= \langle -\sin(t), \cos(t), t \rangle + \vec{C}$$

$$\vec{v}(0) = \langle 0, 1, 0 \rangle + \vec{C}$$

$$\langle 0, 1, 3 \rangle = \langle 0, 1, 0 \rangle + \langle 0, 0, 3 \rangle$$

$$\vec{v}(t) = \langle -\sin(t), \cos(t), 3+t \rangle$$

$$\vec{r}\,'(t) = v(t)$$

$$\vec{r}(t) = \int \vec{v}(t)\,dt + \vec{c}_2$$

$$= \langle \cos(t), \sin(t), 3t + \frac{t^2}{2} \rangle + \vec{c}_2$$

$$\langle 5, 2, 2 \rangle = \langle 1, 0, 0 \rangle + \vec{c}_2$$

$$\vec{c}_2 = \langle 4, 2, 2 \rangle$$

$$\vec{r}(t) = \langle 4 + \cos(t), 2 + \sin(t), 2 + 3t + \frac{t^2}{2} \rangle$$

We reconstruct position from acceleation + two data points (initial position, velocity)

Newton 2:

$\vec{p}$: momentum (total quantity of motion)

$\vec{F}$: force

If object has mass m and velocity $\vec{v}$

$$\vec{p} = m\vec{v} = m\vec{r}\,'$$

The rate of change of momentum is force.

$$\frac{d}{dt}\vec{p} = \vec{F}$$

m constant: $\quad m\vec{r}\,'' = \vec{F} \qquad (\vec{F} = m\vec{a})$

If you know the force acting on an object, you know the acceleration:

$$\vec{a} = \frac{1}{m}\vec{F}.$$

And if you know initial position and velocity then you can reconstruct the position.