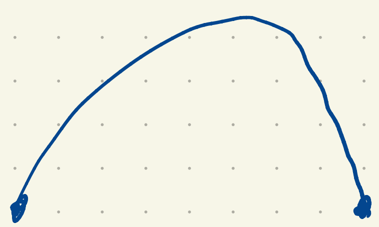
Arclength integrals are a fantastic way to make integrals you can't do exactly.



$$\vec{r}(t) = t\,\hat{i} + (1 - t^4)\,\hat{j}$$

$$-1 \le t \le 1$$

$$|\vec{r}\,'(t)| = \sqrt{1 + (4t^3)^2}$$

$$\int_{-1}^{1} \sqrt{1 + (16)t^6}\,dt$$ hard!

≈ 3.2005

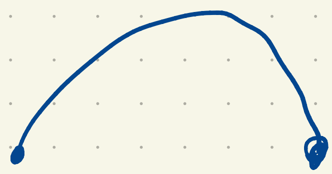
Can still apply Simpson's rule.

$$\vec{r}(t) = 5\cos(t)\,\hat{i} + 5\sin(t)\,\hat{j} \qquad 0 \le t \le \pi$$

parameter t: angle from x-axis.

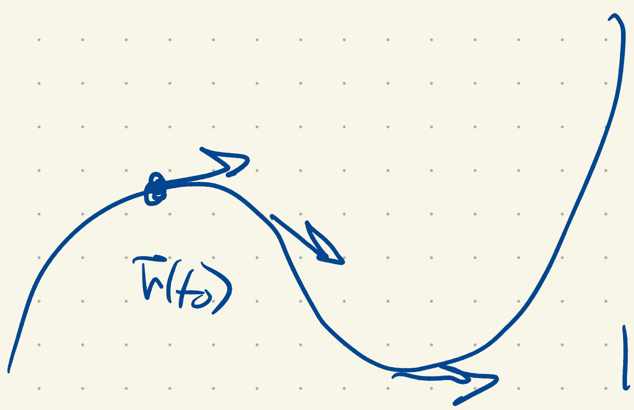
$$\vec{r}(t) = t\,\hat{i} + \sqrt{25 - t^2}\,\hat{j} \qquad -5 \le t \le 5$$

parameterized by x coordinate



We say a curve is parameterized by arclength if

$$|r'(t)| = 1 \text{ at all points.}$$



$|\vec{r}\,'(t)| = 1$ always.

$$\int_{t_0}^{t} |\vec{r}\,'(s)|\,ds = \int_{t_0}^{t} 1\,ds = (t - t_0)$$

$$t = t_0 + \underbrace{\int_{t_0}^{t} |\quad| \, ds}$$

arclength from t_0 to t.

Unit tangent, normal, binormal.

$$\vec{T}(t) = \frac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|} \qquad (\vec{r}\,'(t) \neq 0)$$

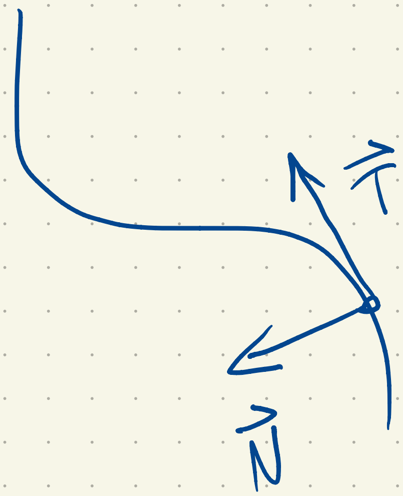
$$|\vec{T}(t)|^2 = 1$$

$$\frac{d}{dt} \vec{T} \cdot \vec{T} = 0$$

$$\vec{T} \cdot \vec{T}\,' = 0$$

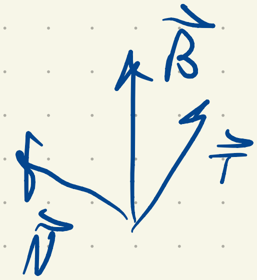
$$\vec{N} = \frac{\vec{T}\,'(t)}{|\vec{T}\,'(t)|}, \text{ unit normal}$$

$(\vec{T}\,' \neq 0)$



3rd vector:

$$\vec{B}(t) = \vec{T}(t) \times \vec{N}(t)$$



(right-handed frame)

The curve carries a frame around with it

(Frenet Serret)

e.g. $\vec{r}(t) = \cos(t)\hat{i} + \sin(t)\hat{j} + t\hat{k}$

$$\vec{r}\,'(t) = -\sin(t)\hat{i} + \cos(t)\hat{j} + \hat{k}$$

$$|\vec{r}\,'(t)| = \sqrt{2}$$

$$\vec{T} = \frac{1}{\sqrt{2}}\left[-s\,\hat{i} + c\,\hat{j} + \hat{k}\right]$$

$$\vec{T}\,' = \frac{1}{\sqrt{2}}\left[-c\,\hat{i} - s\,\hat{j}\right]$$

$$\vec{N} = -\cos(t)\,\hat{i} - \sin(t)\,\hat{j}$$

$$\vec{T} \times \vec{N} = \frac{1}{\sqrt{2}} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -s & c & 1 \\ -c & -s & 0 \end{vmatrix}$$

$$= \frac{1}{\sqrt{2}} \left[+\sin(t)\,\hat{i} - \cos(t)\,\hat{j} + \hat{k} \right]$$