Last class:

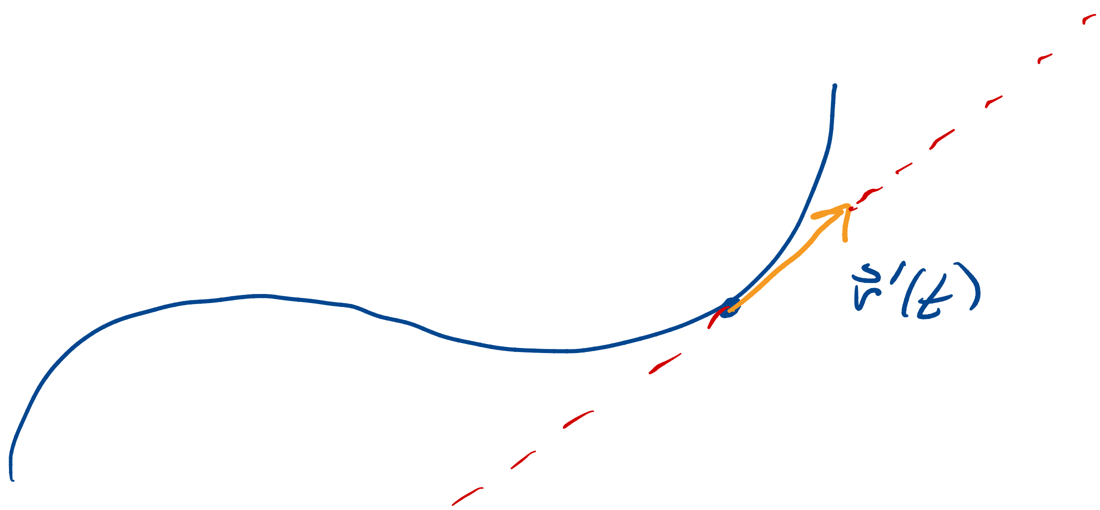
$$\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$$

$$\vec{r}\,'(t) = \lim_{h \to 0} \frac{\vec{r}(t+h) - \vec{r}(t)}{h}$$

$$= \langle x'(t), y'(t), z'(t) \rangle$$

If $\vec{r}(t)$ tells you position at time t,

$\vec{r}\,'(t)$ tells you velocity at time t.



Tangent line: $\vec{r}(t_0) + t\,\vec{r}\,'(t_0)$

$x^2+y^2=z^2$

$\vec{r}(t) = \langle t\cos(t),\ t\sin(t),\ t \rangle$

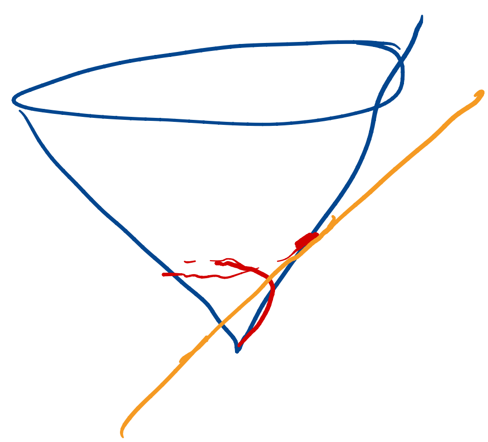
$\vec{r}'(t) = \langle \cos(t) - t\sin(t),\ \sin(t) + t\cos(t),\ 1 \rangle$

$\vec{r}'(2\pi) = \langle 1,\ 2\pi,\ 1 \rangle$

Tangent line: at $t=2\pi$

$\langle 2\pi,\ 0,\ 2\pi \rangle + t\langle 1, 2\pi, 1 \rangle$

$\langle 2\pi + t,\ t\,2\pi,\ 2\pi + t \rangle$



Some rules

$$\frac{d}{dt}\left(\vec{r}(t)+\vec{s}(t)\right) = r'(t) + s'(t)$$

$$\frac{d}{dt}\ \vec{r}(t)\times\vec{s}(t) = \vec{r}\,'(t)\times\vec{s}(t) + \vec{r}(t)\times\vec{s}\,'(t)$$

$$\frac{d}{dt}\ \vec{r}(t)\cdot\vec{s}(t) = \vec{r}\,'(t)\cdot s(t) + \vec{r}(t)\cdot\vec{s}\,'(t)$$

$$\frac{d}{dt}\ \vec{r}(f(t)) = \vec{r}\,'(t)\, f'(t)$$

Suppose $|\vec{r}(t)| = 1$ for all t.

Then $\vec{r}(t)\cdot\vec{r}\,'(t) = 0$.

$$\frac{d}{dt}\ \vec{r}(t)\cdot\vec{r}(t) = \frac{d}{dt}1 = 0$$

$$\vec{r}\,'(t)\cdot\vec{r}(t) + r(t)\cdot\vec{r}\,'(t) = 0$$

$$2\vec{r}(t)\cdot\vec{r}\,'(t) = 0 \quad \checkmark$$

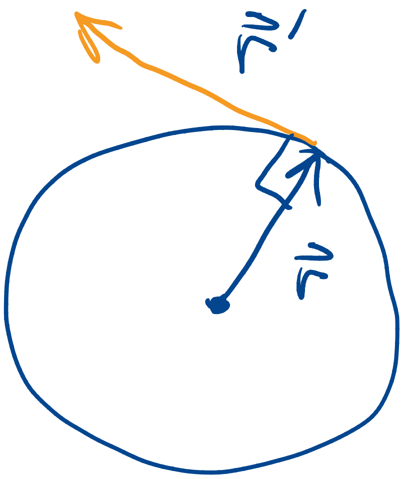
Can replace 1 with any radius

$$\vec{r}(t) = \cos(5t)\,\hat{\imath} + \sin(5t)\,\hat{\jmath}$$

$$\vec{r}\,'(t) = 5\left[-\sin(5t)\,\hat{\imath} + \cos(5t)\,\hat{\jmath}\right]$$

$$\vec{r}(t) \cdot \vec{r}\,'(t) = 5\left(-\cos(5t)\sin(5t) + \sin(5t)\cos(5t)\right)$$

$$= 0 \checkmark$$



Integration. $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle.$

$$\int_a^b \vec{r}(t)\,dt = \left\langle \int_a^b x(t)\,dt, \int_a^b y(t)\,dt, \int_a^b z(t)\,dt \right\rangle$$

(Punchline)

Why? What's it good for?

$$\int_a^b f(x)\,dx \qquad \text{why?}$$

$$\int_1^3 t^3\,dt = \left.\frac{t^4}{4}\right|_1^3 = \frac{81}{4} - \frac{1}{4} = 20$$

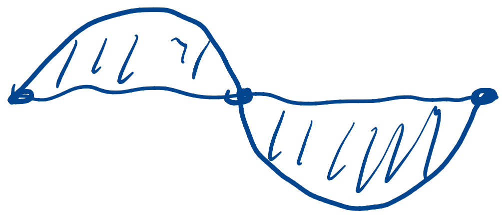
$f(t)$ $\qquad$ $F(t)$

$$F'(t) = f(t)$$

$$\int_1^3 F'(t)\,dt = F(3) - F(1)$$

We integrate to convert a rate of change into a net change.

It's not about areas under curves, really



why is this negative area?

$x'(t) = \sin(t)$

$$\int_0^{2\pi} x'(t)\,dt = x(2\pi) - x(0) = 0 - 0 = 0.$$

net change.

$$\int_a^b \vec{r}\,'(t)\,dt = \vec{r}(b) - \vec{r}(a)$$

$$\to \left\langle \int_a^b x'(t)\,dt,\ \int_a^b y'(t)\,dt,\ \int_a^b z'(t)\,dt \right\rangle$$

$$\left\langle x(b) - x(a),\ y(b) - y(a),\ z(b) - z(a) \right\rangle$$

$= \vec{r}(b) - \vec{r}(a)$ ✓

So if you integrate a velocity $\vec{v}(t)$

$\int_{t_0}^{t_1} \vec{v}(t)\,dt$ is the net displacement (a vector) from time t_0 to time t_1.

E.g. Suppose $r'(t) = \vec{v}(t) = 5\hat{\imath} + (2 - 9.8t)\hat{\jmath}$.

and $\vec{r}(0) = \langle 1, 3, 2 \rangle$

Determine $\vec{r}(t)$.

$$\int_0^t \vec{r}\,'(s)\,ds = \vec{r}(t) - \vec{r}(0)$$

$$\vec{r}(t) = \vec{r}(0) + \int_0^t \vec{r}\,'(s)\,ds$$

$$= \langle 1, 3, 2 \rangle + \int_0^t \vec{v}(s)\,ds$$

$$\int_0^t 5\,ds = 5t, \quad \int_0^t (2 - 9.8s)\,ds = 2t - \frac{9.8t^2}{2}$$

$$\vec{r}(t) = \langle 1 + 5t,\ 3 + 2t - 4.9t^2,\ 2 \rangle.$$

Alt: $\int \vec{v}(t)\,dt := \left\langle \int a(t)\,dt, \int b(t)\,dt, \int c(t)\,dt \right\rangle$

Suppose $\vec{v}(t) = -\sin(t)\,\hat{\imath} + \cos(t)\,\hat{\jmath}$

$\vec{r}(0) = 5\hat{\imath} - 7\hat{\jmath}$

$$\vec{r}(t) = \int \vec{v}(t)\,dt + \vec{C}$$

$$\vec{r}(t) = \cos(t)\,\hat{\imath} + \sin(t)\,\hat{\jmath} + \vec{C}$$

$$\vec{r}(0) = \hat{\imath} + \vec{C}$$