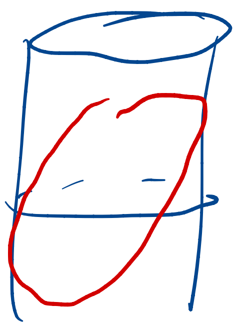
Find a parameterized curve (a.k.a. space curve a.k.a. vector f) describing the intersection of $x^2+y^2=1$ with

$y+z=2$



$$x = \cos(t)$$

$$y = \sin(t)$$

$$z = 2-y = 2-\sin(t)$$

Or

$$x = \cos(5t)$$

$$y = \sin(5t)$$

$$z = 2-\sin(5t)$$

$$\vec{r}(t) = \cos(5t)\hat{i} + \sin(5t)\hat{j} + (2-\sin(5t))\hat{k}$$

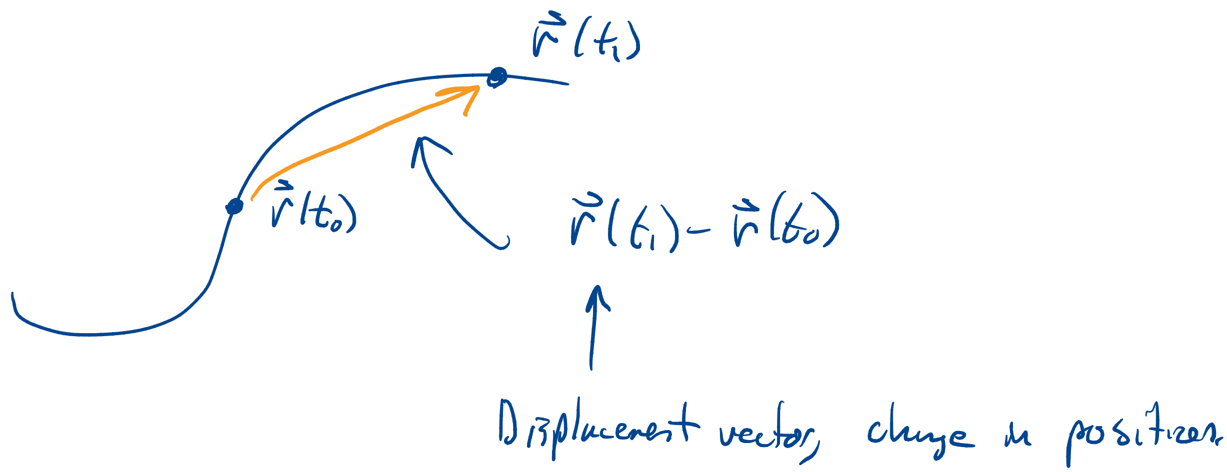
Section 13.2

Derivatives, Integrals of "vector valued functions"

$\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$

"space curve"

If you want, think of it as position $r(t)$ in 3-d space as a function of time.



Change in time: $t_1 - t_0$

Average velocity: $\dfrac{\vec{r}(t_1) - \vec{r}(t_0)}{t_1 - t_0}$

We define $\vec{r}\,'(t) = \lim\limits_{t_1 \to t_0} \dfrac{\vec{r}(t_1) - r(t_0)}{t_1 - t_0}$

This is the derivative of a space curve.

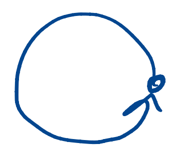
$$\lim_{t_1 \to t_0} \left\langle \frac{x(t_1)-x(t_0)}{t_1-t_0}, \frac{y(t_1)-y(t_0)}{t_1-t_0}, \frac{z(t_1)-z(t_0)}{t_1-t_0} \right\rangle$$

$$\vec{r}\,'(t) = \langle x'(t), y'(t), z'(t) \rangle$$

And you already know how to do the 1-d derivatives

e.g. $\vec{r}(t) = \cos(\omega t)\,\hat{\imath} + \sin(\omega t)\,\hat{\jmath}$

$|\vec{r}(t)| = 1$

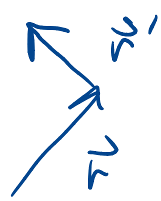


$\vec{r}(0) = \hat{\imath}$

$\vec{r}\left(\frac{2\pi}{\omega}\right) = \hat{\imath}$ period: $2\pi/\omega$

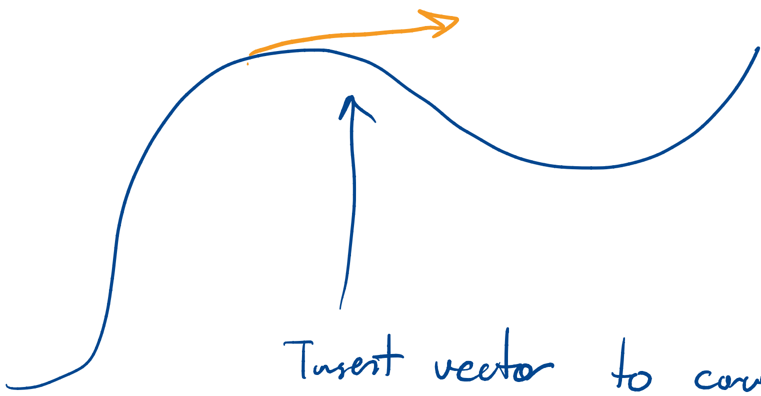
$$\vec{r}\,'(t) = -\omega\sin(\omega t)\,\hat{\imath} + \omega\cos(\omega t)\,\hat{\jmath}$$

$$\vec{r} \cdot \vec{r}\,' = \omega\big(-\sin(\omega t)\cos(\omega t) + \sin(\omega t)\cos(\omega t)\big) = 0.$$



Why does r' grow if ω gets bigger?

How to visualize:



Tangent vector to curve.

It depends on the parameterization (how the curve is traversed).

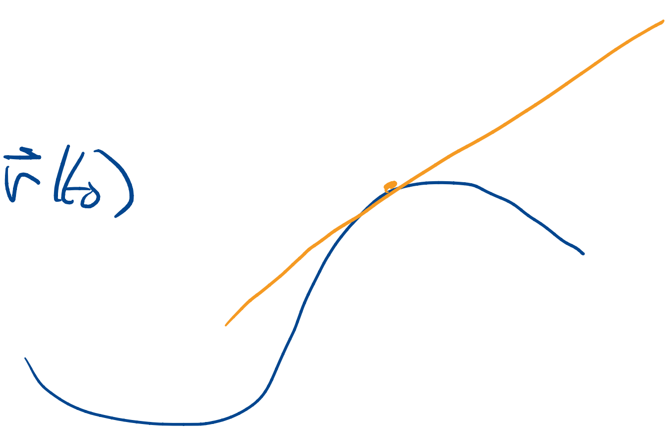
Two related quantities:

$\vec{T}$: unit tangent vector $\dfrac{\vec{r}'(t)}{|\vec{r}'(t)|}$ except if $\vec{r}'(t)=0$.

(ind of param)

Tangent line at $t=t_0$

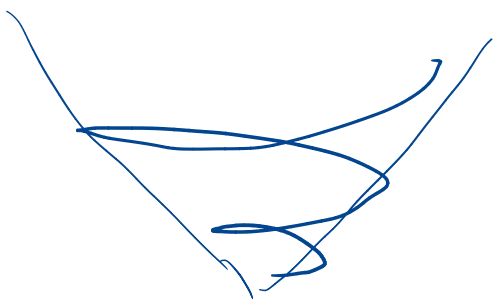
$$\vec{v}(t) = \vec{r}(t_0) + t\,\vec{r}'(t_0)$$



Eg: Find the parametric equation of the tangent line to the curve

$$\vec{r}(t) = \langle t\cos(t), t\sin(t), t\rangle \quad \text{at } t = 2\pi$$

$x^2+y^2=z^2 \Rightarrow$ lies on cone



$$\vec{r}'(t) = \langle -t\sin(t)+\cos(t), t\cos(t)+\sin(t), 1\rangle$$

$$\vec{r}'(2\pi) = \langle 1, 2\pi, 1\rangle$$

$$\vec{l}(t) = \langle 2\pi, 0, 2\pi\rangle + t\langle 1, 2\pi, 1\rangle$$

$$= \langle \underset{x}{2\pi+t}, \underset{y}{2\pi t}, \underset{z}{2\pi+t}\rangle$$

Some rules

$$\frac{d}{dt}\left(\vec{r}(t)+\vec{s}(t)\right) = r'(t) + s'(t)$$

$$\frac{d}{dt}\ \vec{r}(t)\times\vec{s}(t) = \vec{r}'(t)\times\vec{s}(t) + \vec{r}(t)\times\vec{s}'(t)$$

$$\frac{d}{dt}\ \vec{r}(t)\cdot\vec{s}(t) = \vec{r}'(t)\cdot s(t) + \vec{r}(t)\cdot\vec{s}'(t)$$

$$\frac{d}{dt}\ \vec{r}(f(t)) = \vec{r}'(t)\, f'(t)$$

Suppose $\vec{r}(t) = 1$ for all t.

Then $\vec{r}(t)\cdot\vec{r}'(t) = 0$.

$$\frac{d}{dt}\ \vec{r}(t)\cdot\vec{r}(t) = \frac{d}{dt} 1 = 0$$

$$\vec{r}'(t)\cdot\vec{r}(t) + r(t)\cdot\vec{r}'(t) = 0$$

$$2\vec{r}(t)\cdot\vec{r}'(t) = 0 \quad \checkmark$$