E.g. Find the plane thru

$$P = (1,0,2) \quad Q = (-1,3,4) \quad R = (3,5,7)$$

$$\vec{u} = \overrightarrow{PQ} = (-2,3,2)$$

$$\vec{v} = \overrightarrow{PR} = (2,5,5)$$

$$\vec{n} = \vec{u} \times \vec{v} = (15-10)\hat{\imath} - (-10-4)\hat{\jmath} + (-10-6)\hat{k}$$

$$= 5\hat{\imath} + 14\hat{\jmath} - 16\hat{k}$$

$$\vec{n} \cdot (\langle x,y,z\rangle - \langle 1,0,2\rangle) = 0$$

$$5(x-1) + 14y - 16(z-2) = 0$$

$$5x + 14y - 16z = 5 - 32 = -27$$

E.g. Find line of intersection between

$$x+y+z=1 \qquad x-2y+3z=1$$

The direction of line must be normal to $\langle 1,1,1\rangle$ and to $\langle 1,-2,3\rangle$.

$$\begin{matrix} 1 & 1 & 1 \\ 1 & -2 & 3 \end{matrix} \qquad \langle 3+2,\ 1-3,\ -2-1\rangle$$

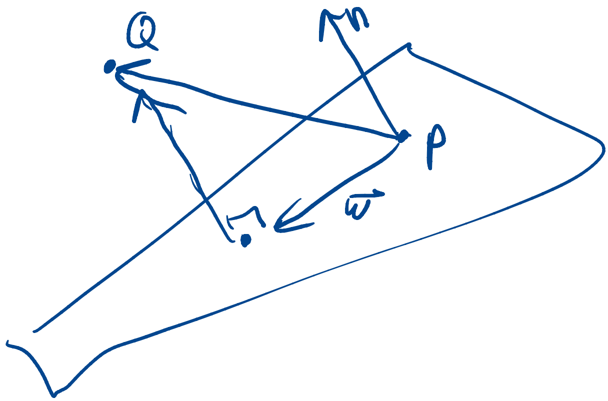
$$\langle 5,-2,-3\rangle$$

$\hookrightarrow$ direction of line!

Need a point. Try $x=0$

$$y+z=1 \qquad y=2/5$$
$$-2y+3z=1$$
$$5z=3 \qquad \langle 0, 2/5, 3/5\rangle$$
$$z=3/5$$

$$\langle 5t, \tfrac{2}{5}-2t, \tfrac{3}{5}-3t\rangle$$



$$\vec{PQ} = a\vec{n} + w$$

dist(Q, plane)

$$\vec{PQ} \cdot \vec{n} = a\,\vec{n} \cdot \vec{n}$$

$$a = \frac{\vec{PQ} \cdot \vec{n}}{\vec{n} \cdot \vec{n}}$$

$$|a\vec{n}| = \frac{|\vec{PQ} \cdot \vec{n}|}{|\vec{n}|}$$

E.g. $Q = (-1, 1, 2)$

$3x - 2y + z = 1$

$n = \langle 3, -2, 1 \rangle \qquad |\vec{n}| = \sqrt{9 + 4 + 1} = \sqrt{14}$

$P: (0,0,1)$

$\vec{PQ} = \langle -1,1,2\rangle - \langle 0,0,1\rangle = \langle -1,1,1\rangle$

$\vec{PQ} \cdot \vec{n} = \langle 3,-2,1\rangle \cdot \langle -1,1,1\rangle$

$= -3 - 2 + 1 = -4$

$\text{dist} = \dfrac{|-4|}{\sqrt{14}} = \dfrac{4}{\sqrt{14}}.$

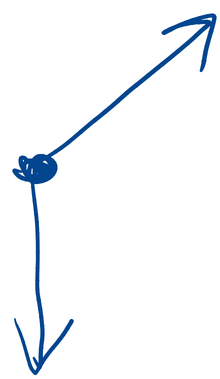
Dist between planes?

Pick a point on one, then dist to the other!

Comments on quiz:

1, 2 generally outstanding.

3: Find the vector of a force of size 500N in the direction $\vec{v} = \langle 1, 2, 1 \rangle$



$\vec{F}$ $\qquad |\vec{F}| = 500N$

a) $\vec{F} = c\vec{v}$ $\qquad \vec{F} = 500N \left\langle \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}} \right\rangle$

$$500N = |\vec{F}| = |c|\,|\vec{v}| = |c|\sqrt{6}$$

$$c = \frac{500N}{\sqrt{6}}$$

Alt:

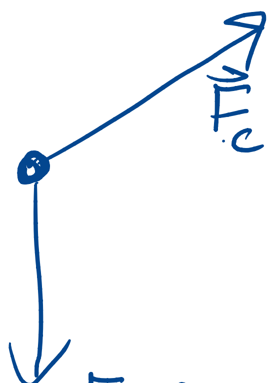
direction $\vec{v} = \langle 1,2,1 \rangle$

unit vector $\vec{u} = \left\langle \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}} \right\rangle$

$500N \cdot \vec{u}$ length: $500N$

direction: same as $\vec{v}$

4)



$\vec{F}_c = \left\langle \frac{500}{\sqrt{6}}, \frac{1000}{\sqrt{6}}, \frac{500}{\sqrt{6}} \right\rangle$

$F_g = \langle 0, 0, -1000 \rangle$

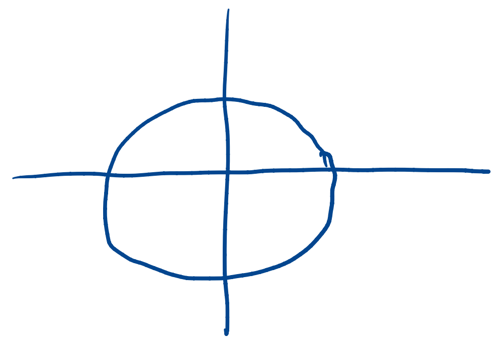
When a body experiences two forces, the net force is the vector sum of the two forces

$$\vec{F} = \left\langle \frac{500}{\sqrt{6}}, \frac{500}{\sqrt{6}}, \frac{500}{\sqrt{6}} - 1000 \right\rangle$$

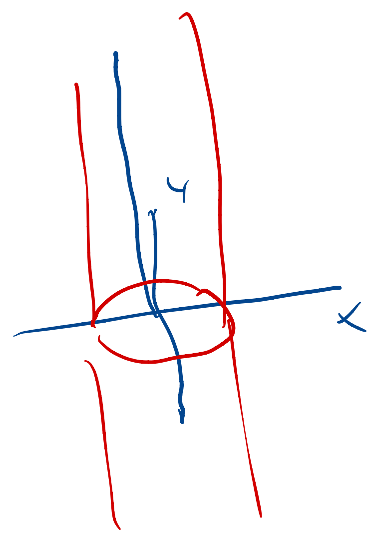
Section 12.6

Elementary surfaces

in $\mathbb{R}^2$ $\quad x^2+y^2=3$

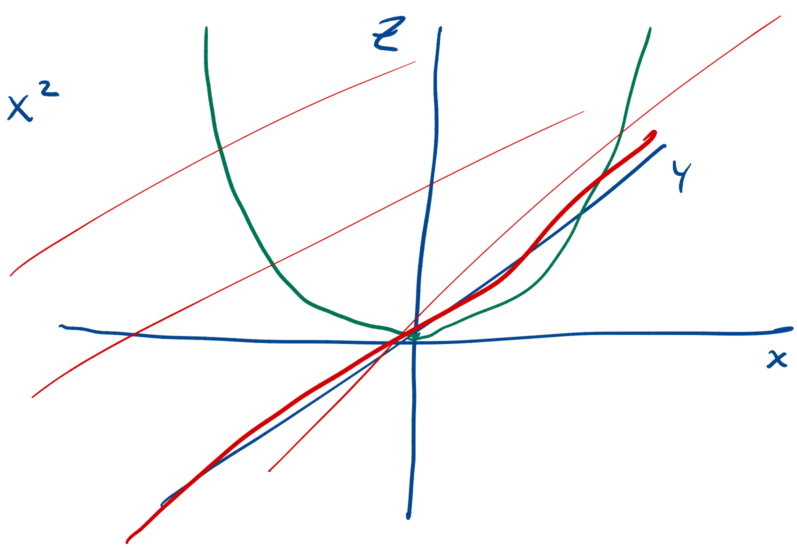


in $\mathbb{R}^3$ $\quad x^2+y^2=3$



This is a cylinder.

$z = x^2$

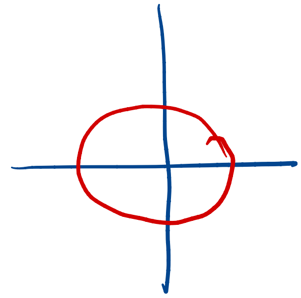


This is also a kind of cylinder

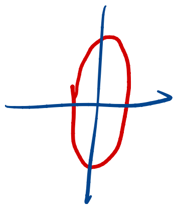
Cylinders are made up from parallel lines

Again in the plane:

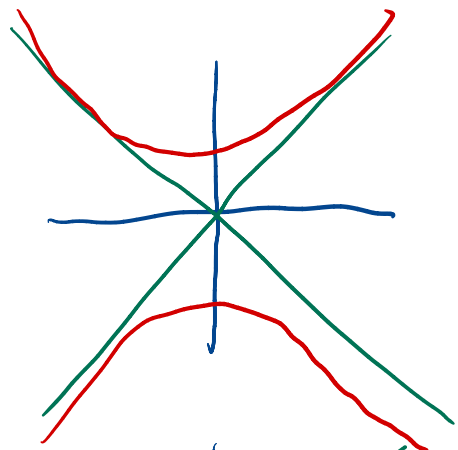
$x^2 + y^2 = 4$



$4x^2 + y^2 = 4$

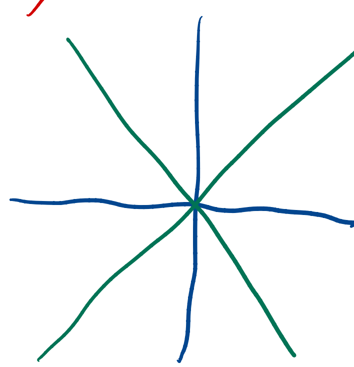


$x^2 - y^2 = 4$

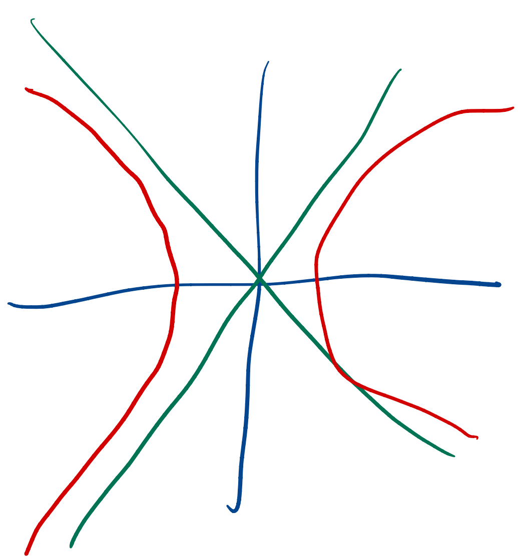


$y^2 = x^2 - 3$

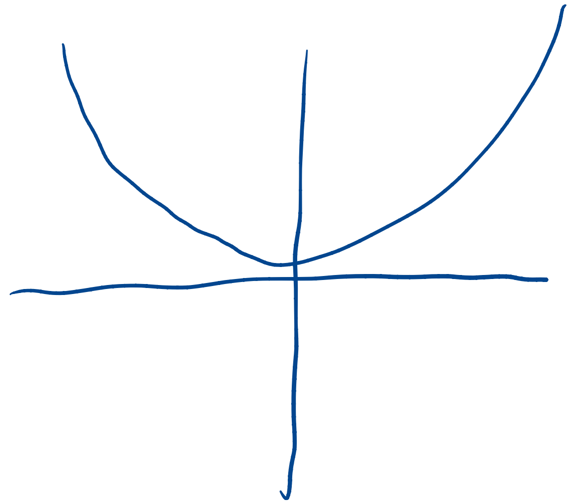
$y^2 = x^2$



$x^2 - y^2 = -3$



$y = x^2$

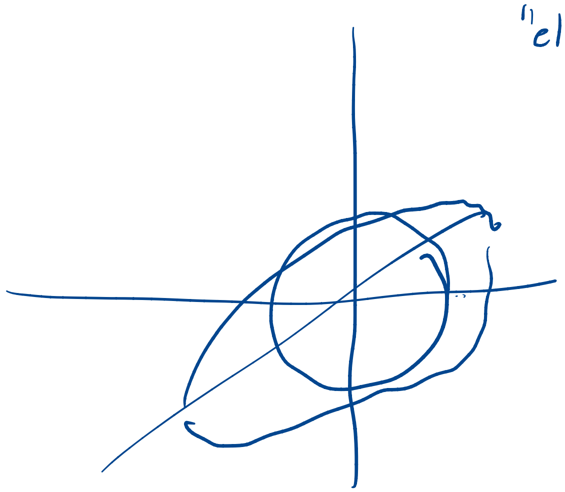


These are all "conic sections"

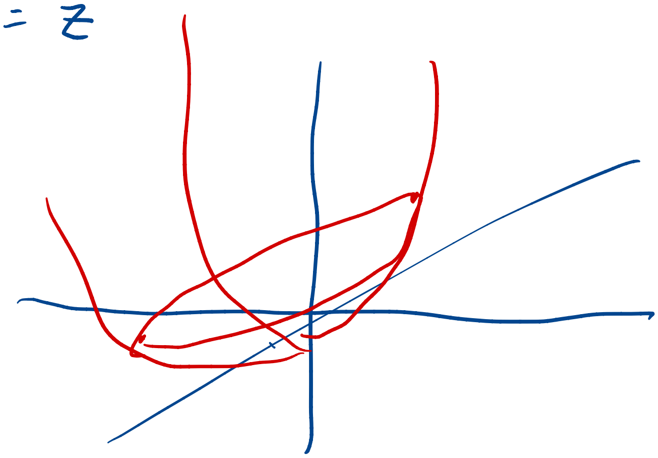
$x^2 + y^2 + z^2 = 4$ sphere of ~~radius~~ radius 4

$4x^2 + y^2 + z^2 = 4$

"ellipsoid"

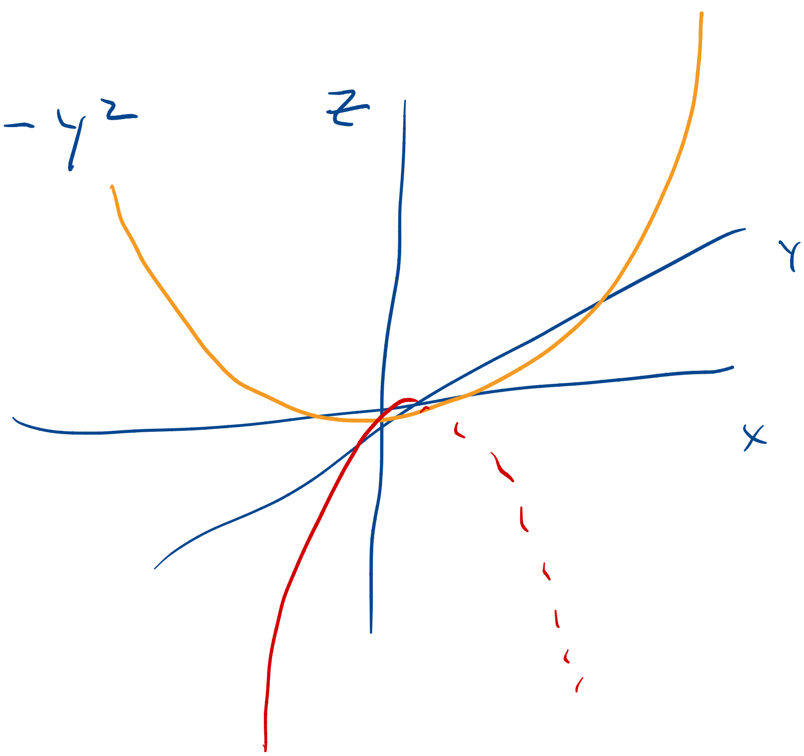


$4x^2 + y^2 = z$



"elliptic paraboloid"

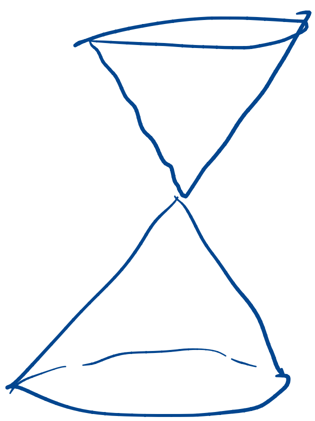
$z = x^2 - y^2$



Saddle: "hyperbolic paraboloid."

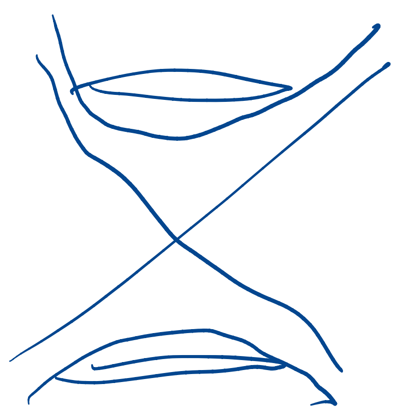
$$z = \sqrt{x^2+y^2}$$

$$x^2+y^2 = z^2$$



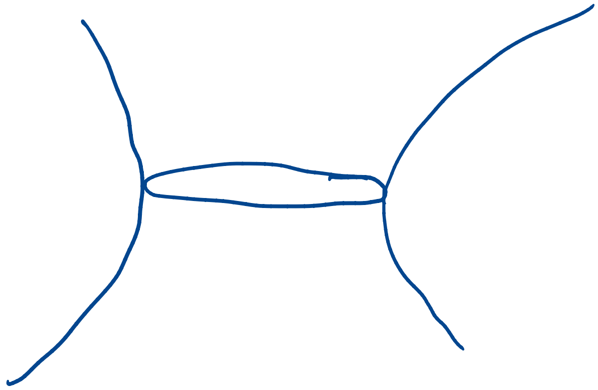
cone!

$$z^2 = x^2+y^2+1$$



hyperboloid of 1. sheet

$$z^2 = x^2+y^2-1$$



$$z^2 = 4x^2 + y^2 - 1$$