Where does this line intersect xy plane?

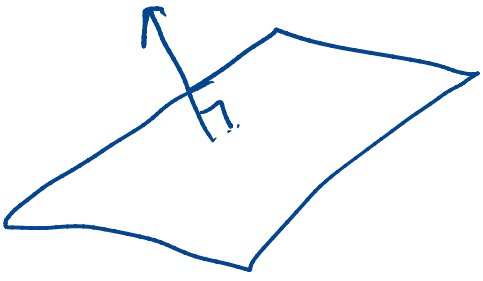
$z = 0 \Rightarrow t = \frac{1}{2}$

$x = 2 + 3 = 5$

$y = -4 + \frac{1}{2} = -\frac{7}{2}$

$(5, -\frac{7}{2}, 0)$

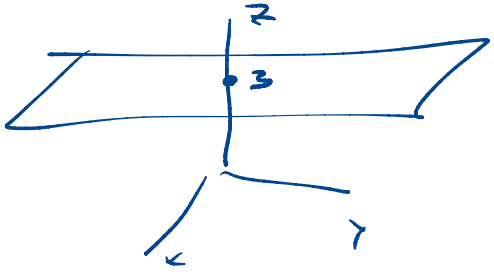
How to describe planes:



In 3-d, every plane has a unique orthogonal direction.

We call a vector ortho to plane a normal vector

$z = 3$



normal vector:

$\langle 0, 0, 1 \rangle$

What is it perp to? n/l the vectors tangent to the plane.

Take any two points in plane, form displacement vector.

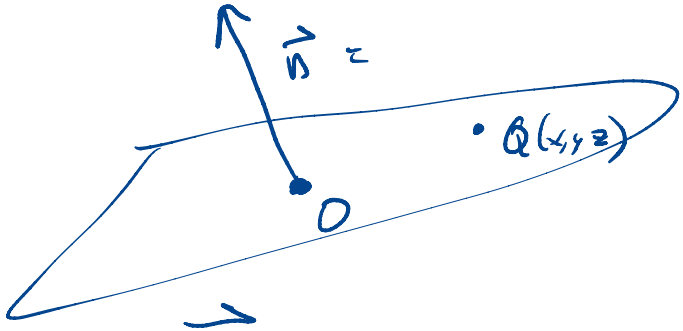
$P(3,5,3) \quad Q(2,-9,3)$

$\overrightarrow{PQ} = (2-3, -9-5, 3-3)$

$= \langle -1, -14, 0 \rangle$

$\langle -1, -14, 0 \rangle \cdot \langle 0, 0, 1 \rangle = 0$ ✓

Planes through the origin:



$$\vec{OQ} = \langle x, y, z \rangle$$

$$0 = \vec{n} \cdot \vec{OQ} = \langle a, b, c \rangle \cdot \langle x, y, z \rangle$$

$$ax + by + cz = 0$$

→ equation of a plane through origin with normal vector $\langle a, b, c \rangle$.

More generally:

$$P(x_0, y_0, z_0) \qquad \vec{PQ} = \langle x - x_0, y - y_0, z - z_0 \rangle$$

$$0 = \vec{n} \cdot \vec{PQ} = a(x - x_0) + b(y - y_0) + c(z - z_0)$$

$$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$$

↳ equation of plane w/ normal

$$\vec{n} = \langle a, b, c \rangle$$

through $P(x_0, y_0, z_0)$

$$ax + by + cz = \underbrace{ax_0 + by_0 + cz_0}$$

$$d$$

$$ax + by + cz = d$$

↳ equation of a plane w/ normal $\langle a, b, c \rangle$.

How many parameters to describe a plane?

two for the direction of normal.

one for which plane three points.

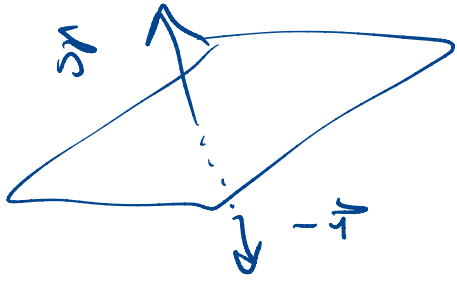
Almost get this by using unit normals

$|\vec{n}| = 1$

$\langle a, b, c \rangle \rightarrow$ 3 terms

$a^2+b^2+c^2 = 1$ down to two terms

But there are two unit vectors



E.g. Find the plane thru

$$P = (1,0,2) \quad Q = (-1,3,4) \quad R = (3,5,7)$$

$$\vec{u} = \overrightarrow{PQ} = (-2,3,2)$$
$$\vec{v} = \overrightarrow{PR} = (2,5,5)$$

$$\vec{n} = \vec{u} \times \vec{v} = (15-10)\hat{i} - (-10-4)\hat{j} + (-10-6)\hat{k}$$

$$= 5\hat{i} + 14\hat{j} - 16\hat{k}$$

$$\vec{n} \cdot (\langle x,y,z \rangle - \langle 1,0,2 \rangle) = 0$$

$$5(x-1) + 14y - 16(z-2) = 0$$

$$5x + 14y - 16z = 5 - 32 = -27$$

E.g. Find line of intersection between

$$x+y+z=1 \qquad x-2y+3z=1$$