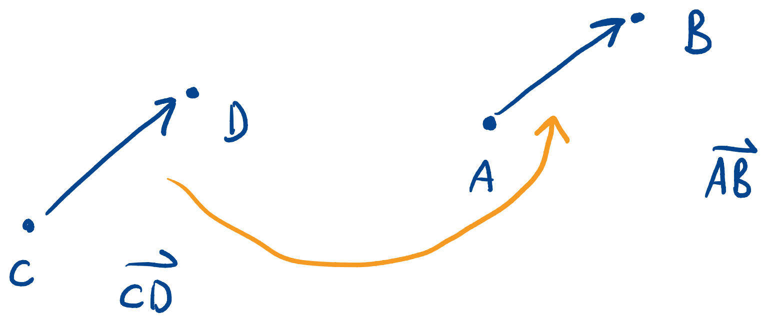
Section 12.2

## Displacement Vectors

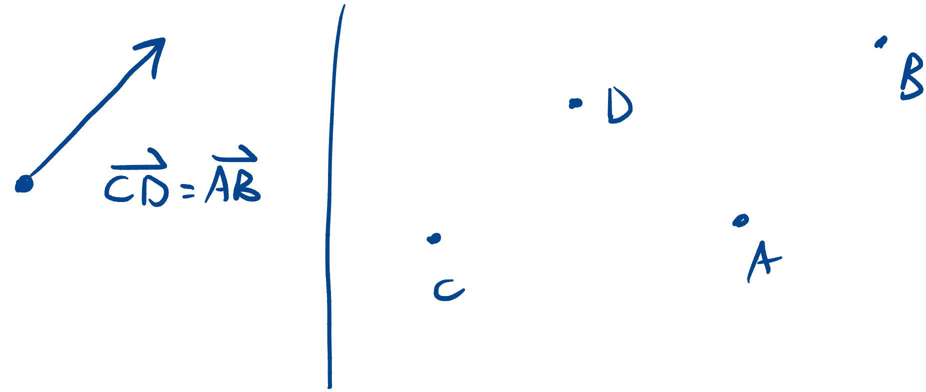



The displacements from C to D and from A to B are the same.

If we translate C to A then D lands on A.

We identify:

$$\overrightarrow{CD} = \overrightarrow{AB}$$




Land of vectors (displacements)

Euclidean space (points)

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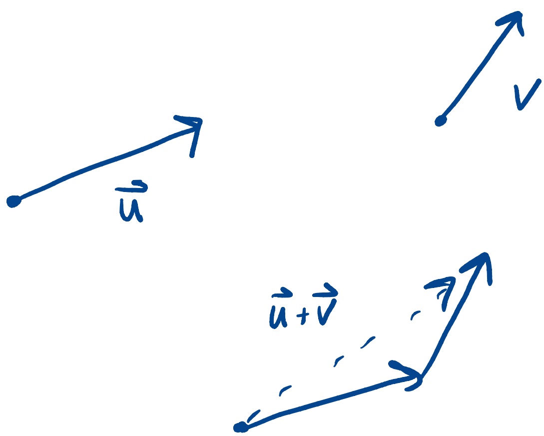
Displacement vectors have a direction (mostly) and a length.

$|\overrightarrow{AB}|$ is just the distance from A to B.

The zero vector does not have a direction.

# Operations on vectors:

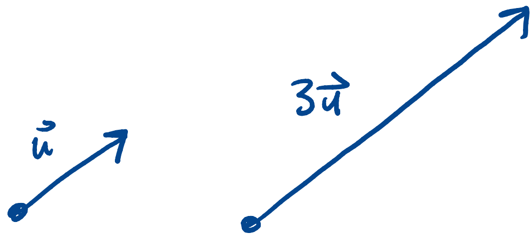
## 1) Vector addition



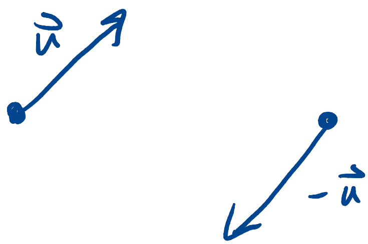

$$\vec{AB} + \vec{BC} = \vec{AC}$$

## 2) Scalar multiplication: $a > 0$, $\vec{u} \neq 0$

$a\vec{u}$ is the vector parallel to $\vec{u}$ with length $a|\vec{u}|$




$a<0$: points in opposite direction




$a \cdot \vec{0} = \vec{0}$ no matter what $a$ is.

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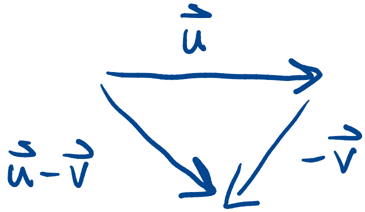
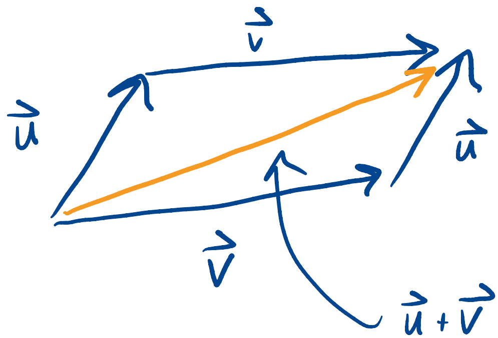
3) Subtraction

$$\vec{u} - \vec{v} = \vec{u} + (-\vec{v})$$




Addition is commutative




The zero vector is special:

$\vec{0}+\vec{u} = \vec{u}+\vec{0}$ no matter what $\vec{u}$ is.

Note: the <u>origin</u> of your Cartesian coordinate system is arbitrary.

The zero vector (zero displacement!) is a very real thing.

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Once you establish Cartesian coordinates, displacement vectors gain coordinates as well:

$P(x_0, y_0, z_0)$

$Q(x_1, y_1, z_1)$

$$\overrightarrow{PQ} = \langle x_1 - x_0, y_1 - y_0, z_1 - z_0 \rangle$$

↳ not standard, but used in text

It's just the difference in coordinates.

The geometric vector operations have very natural algebraic equivalents:

$$\vec{a} = \langle a_1, a_2, a_3 \rangle$$

$$\vec{b} = \langle b_1, b_2, b_3 \rangle$$

$$\vec{a} + \vec{b} = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle$$

$$c\vec{a} = \langle ca_1, ca_2, ca_3 \rangle$$

$$\vec{a} - \vec{b} = \langle a_1 - b_1, a_2 - b_2, a_3 - b_3 \rangle$$

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Properties:

$$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$$

$$c(\vec{a} + \vec{b}) = c\vec{a} + c\vec{b}$$

$$(c+d)\vec{a} = c\vec{a} + d\vec{a}$$

$$c(d\vec{a}) = (cd)\vec{a}$$

$\vec{a} + 0 = \vec{a}$ | $1\vec{a} = \vec{a}$

The length of a vector is the Euclidean length of the displacement

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

$$\left(\sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}\right)$$

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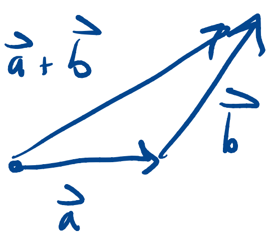
$$|c\vec{a}| = |c|\,|\vec{a}|$$

$$|\vec{a} + \vec{b}| = |\vec{a}| + |\vec{b}| \,?$$ *Nope.*




$$|\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$$

Common operations:

$$\vec{u} = \langle \sqrt{5}, 2, 4 \rangle$$

$$|\vec{u}|^2 = 5 + 4 + 16 = 25$$

$$|\vec{u}| = 5$$

$$\frac{1}{5}\vec{u} = \left\langle \frac{\sqrt{5}}{5}, \frac{2}{5}, \frac{4}{5} \right\rangle$$

$$\uparrow$$

$$\left|\frac{1}{5}\vec{u}\right| = \left|\frac{1}{5}\right| |\vec{u}| = \frac{1}{5} \cdot 5 = 1$$

We say $\frac{1}{5}\vec{u}$ is a unit vector.

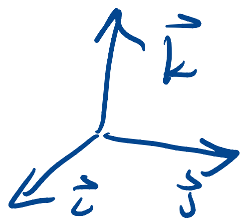
It points parallel to $\vec{u}$ but has unit length.

We give names to three unit vectors that point along the coordinate axes:

$$\vec{i} = \langle 1,0,0 \rangle$$

$$\vec{j} = \langle 0,1,0 \rangle$$

$$\vec{k} = \langle 0,0,1 \rangle$$




(standard basis vectors)

These depend on your coordinates.

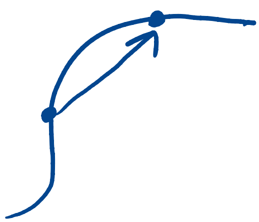
$O$ is special. $\vec{i}$ is <u>not</u>.

$$\vec{a} = \langle a_1, a_2, a_3 \rangle$$
$$= a_1 \vec{i} + a_2 \vec{j} + a_3 \vec{k}.$$

# Other vectorial quantities

- velocity ($m/s$)
- acceleration ($m/s^2$)
- force ($kg\ m/s^2 = N$)

The other parts are "decoration"



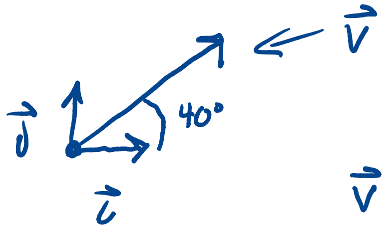
$$\frac{\vec{a}(t_1) - \vec{a}(t_0)}{t_1 - t_0} \qquad \frac{\text{dist}}{\text{time}}$$

send $t_1 \to t_0$ and get an

instantaneous velocity.

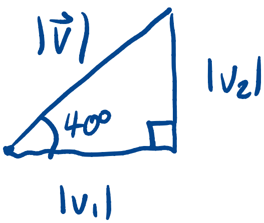
All the rules thus far also apply to these physical variations of displacement vectors

# Components of Physical vectors




$$|\vec{v}| = 10 \text{ km/h}$$

$$\vec{v} = v_1 \vec{i} + v_2 \vec{j}$$




$$|v_1| = |\vec{v}| \cos(40°) \approx 7.7$$

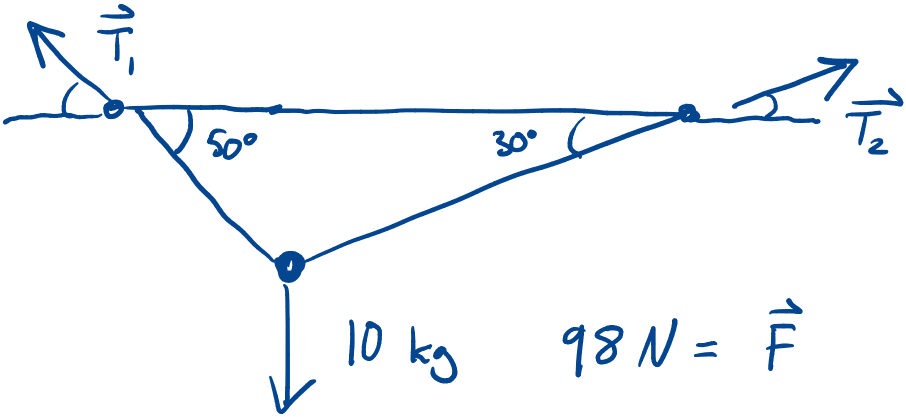
$$|v_2| = |\vec{v}| \sin(40°) \approx 6.4$$

$v_1 > 0$, $v_2 > 0$ also in this case.

$$\vec{v} \approx 7.7 \vec{i} + 6.4 \vec{j}$$

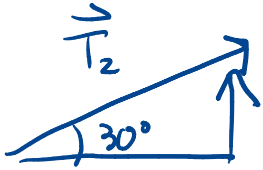
Forces are especially important

Statics: The sum of forces acting on a body add to zero.




$$98\,N = \vec{F}$$

$$\vec{T}_1 + \vec{T}_2 + \vec{F} = 0$$




$$\vec{T}_2 = \cos(30^\circ)\,|T_2|\,\vec{i} + \sin(30^\circ)\,|T_2|\,\vec{j}$$

$$\vec{T}_1 = -\cos(50^\circ)\,|\vec{T}_1|\,\vec{i} + \sin(50^\circ)\,|\vec{T}_1|\,\vec{j}$$

$$\vec{F} = -98\,\vec{j}$$

$\vec{i}$ component:

$$\cos(30^\circ)\,|\vec{T}_2| - \cos(50^\circ)\,|\vec{T}_1| = 0$$

$\vec{j}$ component:

$$\sin(30^\circ)\,|\vec{T}_2| + \sin(50^\circ)\,|\vec{T}_1| = 98$$

$$|\vec{T}_2| = \frac{\cos(50^\circ)}{\cos(30^\circ)}\,|\vec{T}_1|$$

$$\left[\tan(30^\circ)\cos(50^\circ) + \sin(50^\circ)\right]|\vec{T}_1| = 98$$

$$|\vec{T}_1| = 98 \Big/ \underbrace{\left[\tan(30^\circ)\cos(50^\circ) + \sin(50^\circ)\right]}_{1.137\ldots\ = a}$$

$$|\vec{T}_1| = 86.1797\ldots$$

$$|\vec{T}_2| = \frac{\cos(50^\circ)}{\cos(30^\circ)}|\vec{T}_1|$$

$$|\vec{T}_2| = 63.96\ldots$$

$$\vec{T}_1 = -\cos(50^\circ)|\vec{T}_1|\,\vec{i} + \sin(50^\circ)|\vec{T}_1|$$

$$\approx -55.395\,\vec{i} + 66.017\,\vec{j}$$

$$\vec{T}_2 \approx 55.395\,\vec{i} + 31.98\,\vec{j}$$