Section 12.1

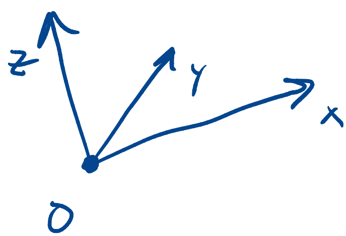
How to make <u>Cartesian</u> coordinates in 3 dimensions

1) pick an origin, O

2) pick a unit distance  (has a notion of distance!)

3) pick 3 mutually perpendicular rays through origin, and label x, y, z

(has a notion of perpendicular!)



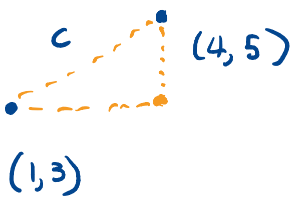
4) The triple $(1, 2, -3)$ encodes the point obtained by

- move in x direction 1 unit
- move in y direction 2 units
- move in $-z$ direction 3 units.

These coordinates are linked to the geometry of 3-d Euclidean space.

There are other systems you might want to use in other applications, but these are a convenient default.

<u>Distance between two points:</u>



$5 - 3 = 2 = b$

$4 - 1 = 3 = a$

$$a^2 + b^2 = c^2$$

$$3^2 + 2^2 = c^2$$

$$9 + 4 = c^2 \Rightarrow c = \sqrt{13}$$

(x_1, y_1)

(x_0, y_0)

$\Delta x = x_1 - x_0$

$\Delta y = y_1 - y_0$

$dist^2 = \Delta x^2 + \Delta y^2$

In 3-d:

(x_1, y_1, z_1)

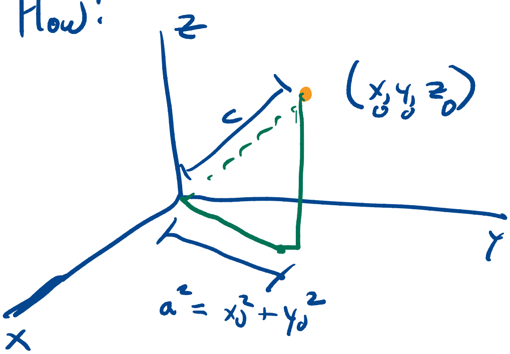
$\Delta z = z_1 - z_0$

(x_0, y_0, z_0)

$dist^2 = \Delta x^2 + \Delta y^2 + \Delta z^2$

$= (x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2$

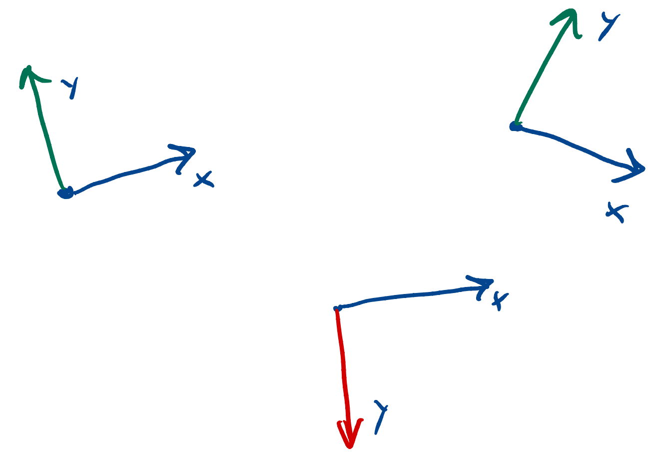
How?

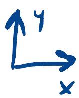


$c^2 = a^2 + z_0^2 = x_0^2 + y_0^2 + z_0^2$

Orientation.

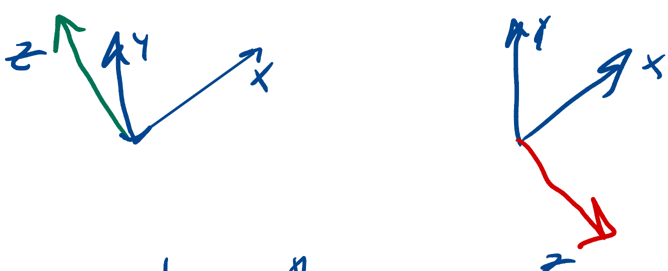
Planes have two classes of cartesian coordinates



Probably the 

ones feel more familiar.

An analogous phenomenon in 3-d.



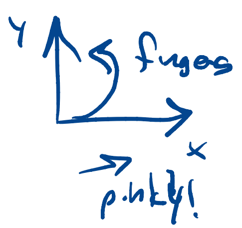
Can't drag one onto other.

If the thing you are coordinatizing has right hands in it, we prefer right-handed coordinate systems.

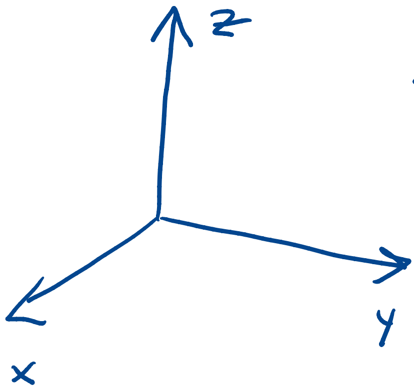
a) use right hand (critical!)

b) lay pinky along positive x-axis

c) rotate hand until fingers curl in direction of positive y-axis



d) thumb points along positive z-axis.



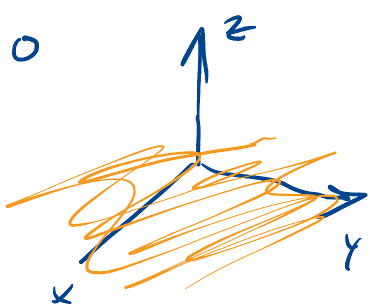
a right handed system

Subsets

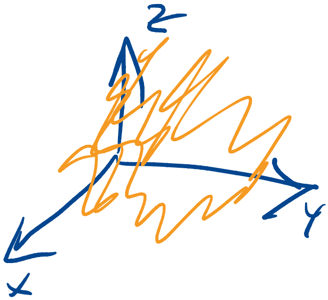
- coordinate planes

xy- plane

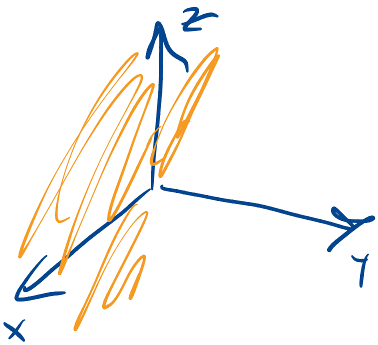
$z = 0$



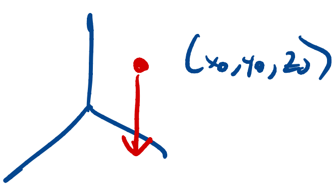
yz- plane $(x=0)$



zx - plane $(y=0)$



(Notation: the projection of (x_0, y_0, z_0) onto the xy-plane is $(x_0, y_0, 0)$)



More soon!

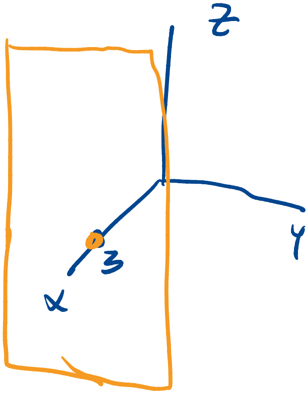
Spheres

The sphere of radius r centered at $P(x_0, y_0, z)$ is the set of points (x, y, z) satisfying

$$(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 = r^2$$

Special planes

$x = 3$ (parallel to yz-plane, passes through $(3,0,0)$)



See text for more

Cylinders, multiple

restrictions.