Bisection

Math 426

University of Alaska Fairbanks

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Last Class

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$$f(x) = c.$$

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$$f(x) = c \Rightarrow f(x) - c = 0$$

$$F(x) = 0$$

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Find functions $F(x)$ that

- A solution of $F(x) = 0$ is $\sqrt{2}$.

✓

$F(x) = x^2 - 2$

$F(x) = x - \sqrt{2}$

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Find functions $F(x)$ that

- A solution of $F(x) = 0$ is $\sqrt{2}$.
- A solution of $F(x) = 0$ is π.

$F(\pi) = 0$

$F(x) = 0$

$x = \pi$

$F(x) = \sin(x)$

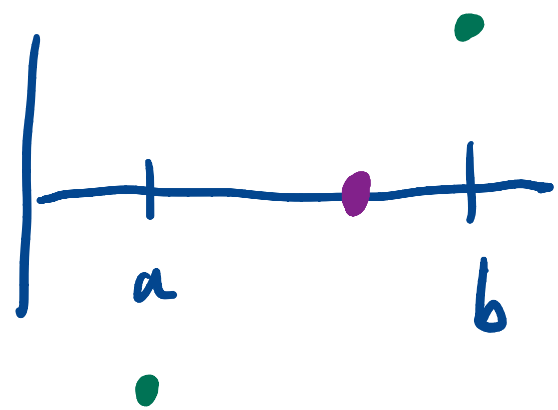
$\sin(\pi) = 0$

Idea of Bisection

Suppose we know numbers a and b with $a < b$ and

$$F(a) < 0$$
$$F(b) > 0$$

Then there should be a c somewhere in the middle so that $F(c) = 0$.



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$$F(a) < 0$$
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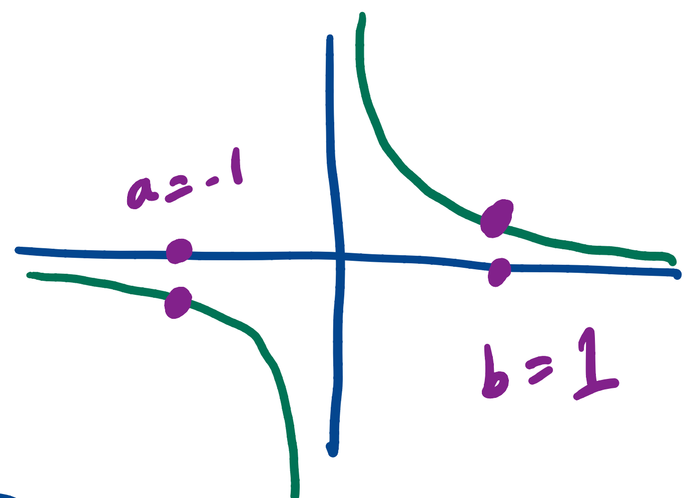
Then there should be a c somewhere in the middle so that $F(c) = 0$.

Not so fast:

- $F(x) = \dfrac{1}{x}$
- $a = -1, \; F(a) = -1$
- $b = 1, \; F(b) = 1$



$$\frac{1}{x} = 0$$

Idea of Bisection

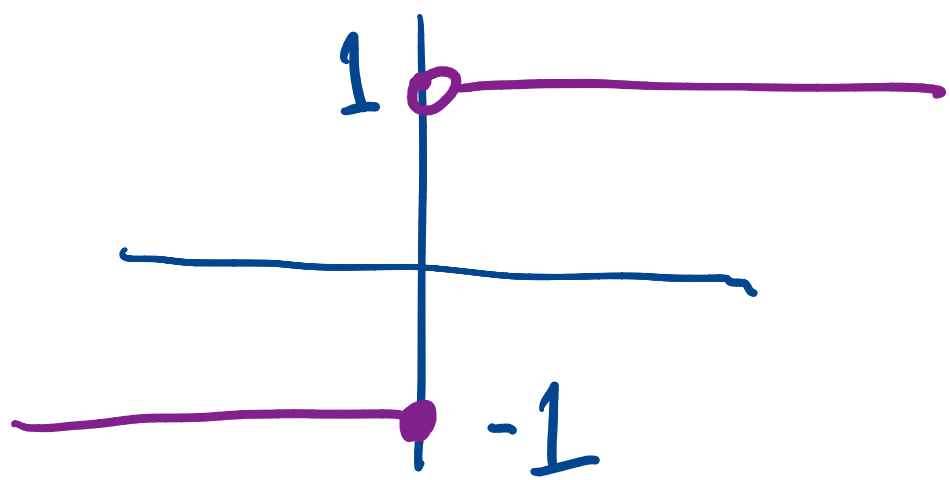
Suppose we know numbers a and b with $a < b$ and

$$F(a) < 0$$
$$F(b) > 0$$

Then there should be a c somewhere in the middle so that $F(c) = 0$.

Not so fast:

- $F(x) = \begin{cases} 1 & x > 0 \\ -1 & x \leq 0 \end{cases}$
- $a = -1,\ F(a) = -1$
- $b = 1,\ F(b) = 1$



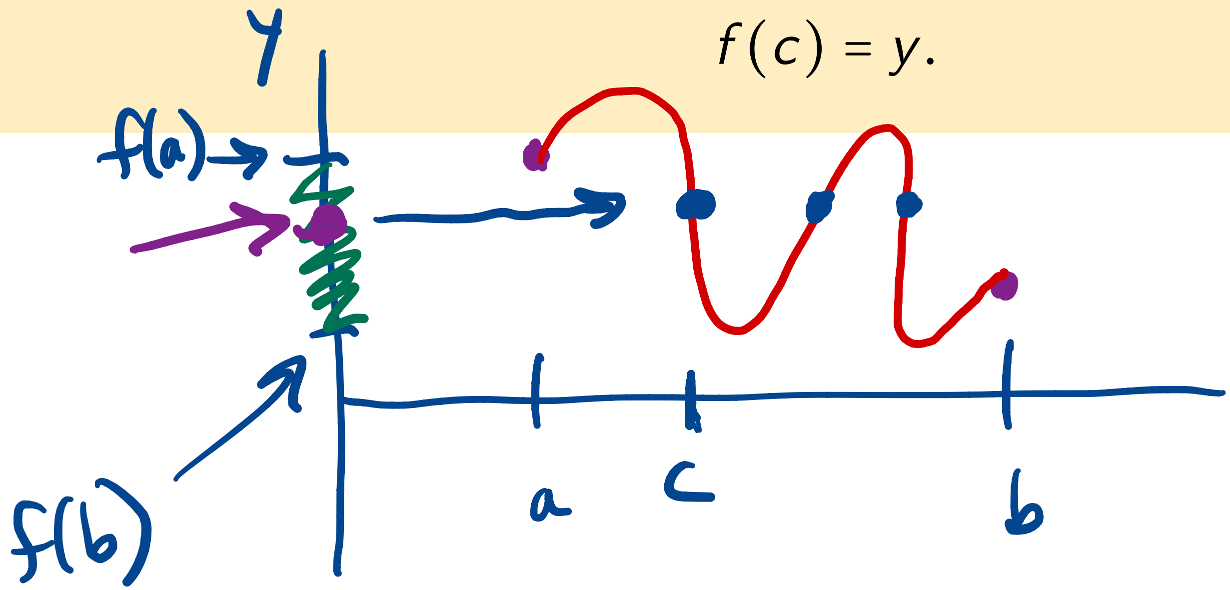
Intermediate Value Theorem

Extra ingredient: **continuity**.

Theorem

Suppose f is a continuous function on an interval $[a, b]$. Then for each value of y between $f(a)$ and $f(b)$ there exists $c \in [a, b]$ such that

$$f(c) = y.$$



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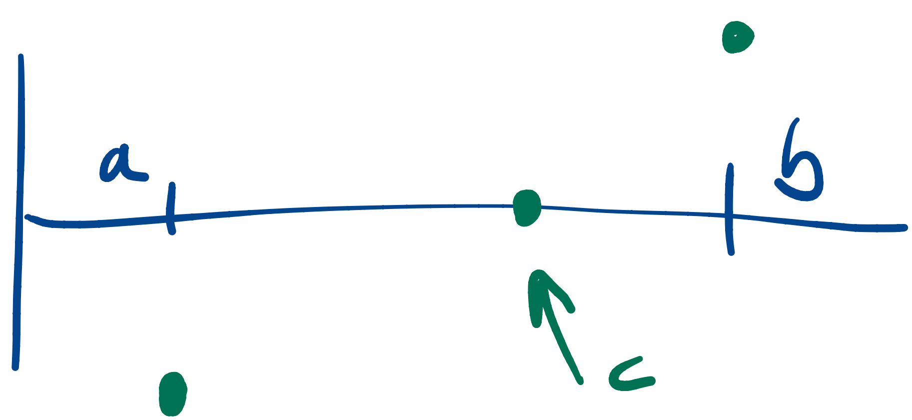

$F(a) < 0$

$F(b) > 0$

we can swap

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So if F is continuous, $F(a) < 0$ and $F(b) > 0$ there is c somewhere in between such that $F(c) = 0$. This guarantees a root.

Bisection Algorithm In A Picture

$m_3 = \frac{a_3 + b_3}{2}$

biggest error $\frac{|b_3 - a_3|}{2} \rightarrow \delta$

midpoint is estimate