

Partial Pivoting

Math 426

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Why we pivot

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}; \quad \mathbf{b} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

Want to solve $A\mathbf{x} = \mathbf{b}$.

Find the LU factorization of

$$\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\mathbf{x} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{aligned} x_1 + x_2 &= 2 \\ x_2 &= 1 \end{aligned}$$

Row exchange by permutation

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$$

$$P = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$PA = \begin{pmatrix} (0,1)A \\ (1,0)A \end{pmatrix} = \begin{pmatrix} \text{second row of } A \\ \text{first row of } A \end{pmatrix}$$

Transform the problem

$$A\mathbf{x} = \mathbf{b}$$

$$\left[\begin{array}{c|c} & \mathbf{b} \\ \hline \mathbf{0} & \mathbf{I} \end{array} \right]$$

$$PA\mathbf{x} = P\mathbf{b}$$

Now do LU factorization:

$$PA = LU$$